Energy dissipation model of particle dampers

M. Saeki

Department of Mechanical Engineering, Shibaura Institute of Technology, Tokyo, Japan, 135-8548

Particle damping is a type of impact damping that is designed to mitigate the response of lightly damped structures under dynamic loading. It consists of granular materials constrained to move between two ends of a cavity in a structure. When attached to a vibrating structure, the collisions of individual particles with cavity boundaries result in a reduction in the vibration amplitude of the structure through momentum transfer. Particle damping with suitable materials can be performed in a wider temperature range than most other types of passive damping. Therefore, it can be applied in extreme temperature environments, where most conventional dampers would fail. Furthermore, it has distinct advantages, such as the use of a robust, simple-to-design and compact system. There is a considerable amount of descriptive data regarding particle damping. However, for a large number of parameters, such as particle size, cavity-filling fractions, material properties and cavity shape, it is extremely difficult to understand the damper performance. There have been some research studies on the development of analytical models to explain the complex phenomenon of particle damping using the discrete element method (DEM). DEM models can be used to simulate the response of dampers, but the prediction of the response is computationally very expensive. This paper presents the results of some numerical analyses in the context of forced vibration in the vertical plane. First, the computational burden of DEM is examined. Then, a new energy dissipation model is represented. The validity of this method is examined by a comparison between experimental and calculated results.

Nomenclature

- $F_p$: component of the contact force of the primary system with the particles in the direction of excitation
- $u$: harmonic displacement of the support point
- $a$: amplitude of the support point
- $\omega$: angular frequency
- $F_N$: normal component of the contact force
- $F_T$: tangential component of the contact force
- $\delta_N$: normal displacement of particle
- $\mu$: coefficient of friction between two particles or between a particle and the cavity wall
- $r$: particle radius
- $t$: center of the particle to the wall
- $k_N$: spring constant
- $E$: modulus of elasticity
- $\nu$: Poisson's ratio
- $L$: distance between the cavity walls that are perpendicular to the direction of the excitation
- $W$: distance between the cavity walls that are perpendicular to the page
- $H$: distance between the right and left cavity walls of the page

I. Introduction

For mitigating the response of lightly damped structures, there are many passive damping devices. Oil and viscoelastic dampers are typical damping devices that are widely used. However, the efficiency of these dampers
depends on various environmental conditions. Particularly, these dampers cannot be applied in extreme temperature environments. Particle damping can be performed in a wider temperature range than most other forms of passive damping. The dampers used for this technique have a simple structure that consists of impactors, such as granular materials, operating in the cavity of a primary system. The principle behind particle damping is the removal of vibratory energy through losses that occur during impact of granular materials.

Many experimental and analytical studies have demonstrated the effectiveness of particle dampers\(^4\)\(^-\)\(^10\). Araki et al.\(^3\) investigated the characteristics of particle dampers in a single-degree-of-freedom system that was subjected to an external sinusoidal force. Their theoretical analysis has focused on the concept of an equivalent single-particle damper. Papalou and Masri\(^7\) studied particle dampers in a horizontally vibrating system under random excitation. They investigated the effects of mass ratio, particle size, cavity dimensions and excitation level on the performance of the system experimentally. Saeki\(^9\)\(^,\)\(^10\) investigated the effects of system parameters on the damping performance by the discrete element method (DEM)\(^11\). This method makes it possible to consider the effects of granularity such as size and number of particles. However, the high computational burden of DEM is a concern. DEM does not seem to be widely used.

This paper presents the computational burden of DEM analysis. In addition, a new method for estimating an energy dissipation model is presented. The model is the approximated contact force of particles to the primary system as a function of damping and stiffness terms. The validity of this method is examined by a comparison of experimental results. It is shown that the mass ratio affects the damping and stiffness terms.

II. Particle damper

Figure 1 shows a model of a particle damper. In this figure, the x-z plane is parallel to the horizontal plane. The primary system consists of a mass M, a spring k, and a damper with a damping constant c, which is assumed to move only along the direction of the y-axis. The primary system is excited by the motion of the support point U. When granular materials are placed in the cavity of the primary system, as shown in Fig. 1, the collision of the particles with the wall of the cavity results in an exchange of momentum and energy dissipation, attenuating the motion of the primary system. The equation of motion for the primary system is expressed as

\[
F_p = M\ddot{y} + c(y - \dot{u}) + k(y - u)
\]

where the dots denote time derivatives.

III. DEM

The numerical method used in this study is based on DEM. Figure 2 shows the contact state between the two particles i and j. In this figure, P and C denote the center of a particle and the contact point, respectively. As shown in Fig. 2, the contact force can be divided into normal N and tangential T components. The normal component \(F_N\) of the contact force is modeled by the sum of the spring force based on the Hertzian contact theory and the damping force, and is expressed by

\[
F_N = k_N \delta_N^{1/2} + c_N \delta_N^{1/2} \dot{\delta}_N
\]

The tangential component \(F_T\) of the contact force is given in Eq. (3), considering Coulomb’s law of friction.
In cases of interparticle contact and particle contact with the wall, normal displacement is given by Eqs. (4) and (5), respectively.

\[ \delta_n = 2r - |\hat{p}_j - \hat{p}_i| \]  

(4)

\[ \delta_n = r - s \]  

(5)

On the basis of the Hertzian theory of elastic contact for spheres, the spring constant in Eq. (2) is expressed as

\[ k_y = \frac{\sqrt{2r}}{3} \frac{E_p}{(1-v_p^2)} \]  

(6)

In the case of contact between a sphere and the wall, the spring constant is expressed as

\[ k_y = \frac{4\sqrt{r}}{3} \frac{E_pE_0}{(1-v_p^2)E_0 + (1-v_0^2)E_p} \]  

(7)

where the subscripts 0 and p denote the wall and a particle, respectively.

The motions of all particles can be analyzed using Eqs. (2)-(7).

IV. Experimental results

Figure 3 shows the experiment apparatus used in this study. The primary structure is made of acrylic resin and supported by two leaf springs. The primary system was excited at the support of the structure using a perpendicularly vibrating shaker. Two accelerometers were used to measure the motions of the primary structure and shaker. Acrylic resin balls of uniform size are used as impactors. The equivalent properties of this system were found to be \( M = 0.293 \) kg, \( c = 0.116 \) Ns/m and \( k = 1602.7 \) N/m. The dimensions of the cavity are \( L = 58 \) mm, \( H = 38 \) mm and \( W = 38 \) mm.

Figure 4 shows a comparison between experimental and calculated results. This figure shows the plot of the root mean square (rms) of the velocity amplitude of the primary system versus the frequency \( f \) of the vibrating support ( \( f = \omega / 2\pi \) ). In this figure, \( \lambda \) is the mass ratio, which is the total mass of granular materials per mass of the primary system. Although the calculated results do not reproduce the experimental results completely, the
calculated results show an experimental trend. Therefore, DEM is an effective numerical method for estimating the performance of particle damping.

Figure 5 shows the relationship between the computational time and the frequency. The computational conditions used in this figure are the same as those in Fig. 4. It is clear that the computational time is large in the case of a large mass ratio. The possible reason for this is that the possibility of interparticle contact and particle contact with the wall increases when the number of particles is large. For $\lambda = 0.05$, the computational time is independent of the frequency. This trend is different from that for $\lambda = 0.15$. For $f < 11\text{Hz}$ or $f > 13\text{Hz}$, the computational time is larger than those for other frequencies. The possible reason for this is as follows. As shown in Fig. 4, the velocity amplitude is smaller for $f < 11\text{Hz}$ or $f > 13\text{Hz}$ than for other frequencies. Thus, many particles easily to collide with other particles or the wall of the cavity for $f < 11\text{Hz}$ or $f > 13\text{Hz}$, it results in a high computational burden. Since the computational time takes over 6 hours for $\lambda = 0.15$, it is clear that the computational burden of DEM is much high. Therefore, to estimate the efficiency of particle damping easily, it is necessary to introduce an effective algorithm or to develop another method for estimating it.

V. Simplified approach for particle damping

In this study, a new energy dissipation model is suggested. For every experimental result obtained in this study, the relationship between the velocity of the primary structure and time was observed as a sinusoidal curve. Therefore, the velocity of the support point is approximated by

$$\dot{y} = v_{y_{max}} \sin(\omega_f t - \theta_f)$$

(8)

Figure 6 shows the time history for the primary system velocity and approximated points obtained from Eq. (8). It is clear that the approximated results show an experimental trend with reasonable accuracy. Although the time history of the acceleration for the primary system is nonlinear, the acceleration and displacement for the primary system are expressed in the following forms for simplicity:

![Figure 6. Time history of velocity](image)

![Figure 7. $F_p$-relative velocity](image)
Then, the contact force can be expressed in the following form using Eqs. (8)- (10):

\[ F_p = C_p (\dot{y} - \dot{u}) + K_p (y - u) \]  

(11)

Figure 7 shows the relationship between the contact force and relative displacement \((y - u)\) obtained from Eq. (11). The area inside the ellipse denotes the energy dissipation. In this figure, the data points indicated by circles are approximated using

\[ C_p = C_{p0} + \omega^2 + C_\omega/\omega^0 \]
\[ K_p = K_{p0} + \omega^2 + K_\omega/\omega^0 \]  

(12)

Figures 8 and 9 show the damping coefficient and stiffness coefficient versus the excitation frequency obtained from Eq. (12), respectively. In these figures, solid lines indicate the approximated data points. As the mass ratio \(\lambda\) increases, the damping coefficient increases and the stiffness coefficient decreases. In addition, introducing the approximated Eq. (12) to Eq. (1), the velocity amplitude of the primary system is expressed as

\[ Y = \frac{\alpha \omega}{\left(1 - \left(\frac{\omega}{\omega_p}\right)^2 \right)^{1/2} + \left(2 \gamma_p \frac{\omega}{\omega_p}\right)^2} \]  

(13)

Figure 10 shows a comparison between the experimental and calculated results obtained using Eq. (7). Both this figure and Fig. 4 have the same ordinate and abscissa. The calculated results reproduce the experimental results. Therefore, the approach used in this study is effective for estimating the energy dissipation of particle damping.

VI. Conclusion

The performance of particle damping was investigated analytically. First, the computational burden of DEM was investigated. It was found that the computational time increases as the mass ratio increases. Then, a new method for estimating an energy dissipation model was presented. The model is the approximated contact force of particles to
the primary system as a function of damping and stiffness terms. The validity of this method was examined by a comparison of experimental results. It was shown that the mass ratio affects the damping and stiffness terms.

References