Non-Linear Viscoelastic Analysis and the Design of Super-Pressure Balloons: Stress, Strain and Stability

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The application of geometric and materially non-linear finite element analysis techniques to the NASA Super-Pressure Balloon Program has been driven by the need to understand and overcome deployment and stability problems that have shadowed the chosen ‘pumpkin’ design. Early iterations of the super-pressure balloon designs showed problems of shape instability, characterized by improper deployment and the potential for overall geometric instability once deployed. The latter has been reproduced numerically using inTENS, and the former are better understood following a series of large-scale hangar tests simulating launch and ascent. In both cases the solution lies in minimizing the film lobing between the tendons. These tendons, which span between base and apex end fittings, cause the characteristic pumpkin shape of the balloons and also provide valuable constraint against excessive film deformation. There is also the requirement to generate a biaxial stress field in order to mobilize in-plane shear stiffness. Achieving this will test the structural performance of the film to its limits and make full use of the strain arresting feature of the much stiffer tendons. A full numerical model that takes account of the non-linear viscoelastic response of the film is required to properly judge these design issues over the whole duration of a flight. Stress concentrations must be considered, along with the influence of shape change through film creep on the maintenance of stability. This paper summarizes the current numerical approach, and describes the implementation of ‘whole flight’ analyses of balloon stresses and stability.

I. Introduction

Tensys have a long-established background in the shape generation and load analysis of architectural stressed membrane structures. Founded upon their inTENS finite element analysis suite, these activities have broadened to encompass ‘lighter than air’ structures such as aerostats, hybrid air-vehicles and stratospheric balloons. Since 2004 Tensys have acted as consultants to the NASA Super-Pressure Balloon Program.

With the recent successful conclusion of the record breaking test flight 591NT16, a 7mcf super-pressure balloon flight that was terminated after 54 days over Antarctica, design focus is placed firmly on the balloon performance over long duration flights. A number of design requirements that impact upon analysis capabilities have evolved as the project has proceeded:

1) Resistance to overall geometric instability requires the use of biaxially stressed film lobes in order to mobilize the distortion resistance of shear stiffness.

2) Tendons should be axially constrained to the balloon film rather than free to slip within their sleeve. This prevents film migration towards the polar region and consequent reduction in overall stability

3) Mobilizing the film structurally in both hoop and meridional directions has the effect of introducing thermal effects due to the differential CTE between film and tendon. This has the effect of inducing additional relatively high meridional film stresses.

4) The solution to the S-cleft deployment instability is a further minimization of excess material through flattening of the gores. Again higher film stresses will result.

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The resulting design will be inevitably be a balance between maximizing film stresses and minimizing excess material. It will be essential to consider the non-linear viscoelastic response of the polyethylene film in the design. Equivalent elastic properties based upon a ‘point in time’ assessment of the viscoelastic response have been employed. Using the full viscoelastic representation in a sequence of equilibrium analyses that represent successive time steps in the sequence of deployment and flight will enable:

1) Allowance for stress relaxation over time. This is relevant to both the overall film behavior and local effects due to fabrication tolerances.

2) Determination of the balloon geometry change over time due to creep. The overall geometric stability of the balloon must be checked at interim and final stages of a flight to ensure that increasing size does not bring about a critical condition.

Figure 1. Flight 591NT Antarctica January 2009

Long duration super-pressure balloon design requires an analysis approach that addresses the questions of stress and stability over the duration of a flight by time stepping analyses using an appropriate material model. This paper summarizes the Dynamic Relaxation approach to stress and stability analysis inherent in inTENS, and focuses in particular on:

1) Implementation of an alternative application of the Incremental Schapery Rand (ISR) representation of the non-linear viscoelastic response of the polyethylene balloon film. This is based upon the relaxation modulus, rather than the creep compliance, and as such fits more efficiently into the Dynamic Relaxation analysis procedure used within inTENS. Comparisons of results between the two approaches are given.

2) Verification of the material model by comparison with material tests.

3) Verification of the application to pumpkin balloon structures by comparison with scale model tests.

4) Application of inTENS with ISR to time-stepping analyses of a balloon flight including diurnal variations of temperature and pressure. This includes the demonstration of a method for checking the likelihood of overall instability developing at any particular time in the flight as both balloon geometry and film properties change due to viscoelastic effects.
II. Numerical Modeling

The use of stressed membrane structures was pioneered by Frei Otto\(^1\) for architectural applications focused upon the use of natural forms for optimization of material usage. In parallel Walter Bird and others were developing inflatable membrane structures as enclosures for radar installations on the DEW line and elsewhere.

![Membrane Structures](image)

**Figure 2. Membrane Structures**

The early engineering work on these structures was based upon physical models for both load estimation and the generation of the membrane and cable cutting geometry necessary for fabrication. Geometrically non-linear structural analysis was undertaken for the cable-net pavilions of the 1972 Olympic Games in Munich. However large scale physical models were needed to provide initial geometry of sufficient accurate to enable convergence of these analyses.

The shape of a stressed membrane surface is the key to its engineering, determining both the magnitude and orientation of stresses and deflections under load. The design process typically involves progressive iterations of shape and analysis. The need to determine and refine initial equilibrated shapes, coupled with significant geometric non-linearity, led to the development of specialist finite element software for membrane structure design. The numerical techniques most suited to shape or form generation have typically been adapted to provide the necessary load analysis capability.

A. Dynamic Relaxation

The inTENS finite element program suite for stressed membrane structure design has been under continual development by Tensys since 1990. It is based upon the Dynamic Relaxation (DR) solution technique that has particular advantages for this class of problem. Program modules are available for model generation, stress controlled equilibrium shape determination, load analysis, and membrane patterning\(^2\).

The static solution of both linear and non-linear structures subject to load may be regarded as the limiting equilibrium state of damped structural vibrations excited by that load. The physical basis of Dynamic Relaxation was initially perceived as the step-by-step solution, for small time increments $\Delta t$ of Newton’s Second Law of Motion applied to a loaded structure subjected to an imposed viscous damping.
The basic iterative equations for the motion any node in space at time \( t \) are obtained directly from Newton (Force=Mass x Acceleration):

\[
R^t = MA^t
\]

which may be expressed in central difference form

\[
R^t = M \left( V^{t+\Delta t/2} - V^{t-\Delta t/2} \right) / \Delta t
\]

giving the recurrence relation for nodal velocities:

\[
V^{t+\Delta t/2} = V^{t-\Delta t/2} + \Delta t \frac{R^t}{M}
\]

where
- \( R^t \) is the node residual force at time \( t \)
- \( V^t \) is the associated node velocity
- \( M \) is the node mass and \( \Delta t \) the time interval

The residual forces \( R^t \) were computed for the then current node co-ordinates \( X^t \). An updated set is calculated from the incremented node coordinates:

\[
X^{t+\Delta t} = X^{t+\Delta t} + \Delta t V^{t+\Delta t/2}
\]

The original need to compute an optimized viscous damping coefficient has been eliminated by the adoption of a kinetic damping procedure. When an oscillating body passes through a local approximation to its static equilibrium position, then the plot of total kinetic energy against time passes through a local maximum. The total kinetic energy is traced as undamped iterations proceed, and all current node velocities are reset to zero whenever an energy peak is detected. The analysis continues, progressively eliminating the kinetic energy from various modes of vibration until the required degree of convergence is obtained. The process is automatic, requires no specification of damping factors, and can handle gross out of balance forces without the need for additional constraints. This means that gross geometrical changes and stiffness modifications may be accommodated during an analysis.

On first inspection this might seem a rather long-winded approach, but it does have significant advantages when applied to the design of tension structures:

1) An explicit solution technique – a time-stepping dynamic analysis with automated kinetic energy damping control. The static solution to an applied loading case is obtained by damping out a dynamic application of that load.

2) A natural treatment of large deformations coupled with a tolerance of the very large out-of-balance forces that are often associated with significant geometric non-linearity.

3) Computer storage requirements increase linearly with problem size, and the computations are all held within memory rather than requiring significant interaction with storage devices. Surprisingly large problems may be tackled on conventional PC hardware.

4) Needing only individual element natural stiffnesses without the formation of an overall structure matrix it is straightforward to introduce on-off non-linearities such as cable slackening and membrane wrinkling. Similarly a wide range of nodal constraints is possible. Specified element stress states may also be introduced, such as for the shape generation of membrane surfaces where the final geometry may be adjusted by continually varying stresses in the fabric weave directions.

5) A full range of elements is available to facilitate the modeling of complete structures, including membranes, cables, struts and beams.

6) By using large numbers of simple elements within a method founded on a clear physical analogy, the interpretation of results and unforeseen occurrences is facilitated.
Note that
1) An overall stiffness matrix is not required.
2) Checks on membrane wrinkling are applied individually at stage B2.
3) Non-linear stress strain relations are introduced by revised code for stage B2.
4) Geometric non-linearity and rigid body movement handled automatically as all strain calculations and element force resolution based upon current updated coordinates.
5) Time steps are fictitious and node masses determined to optimize convergence. Kinetic energy damping results in a static solution to particular applied load state.

Figure 3. Dynamic Relaxation Flowchart

Although initially developed for fabric structure applications\(^3\), the program has been progressively extended for broader applications. Recent applications include large kites, such the SkySails system for wind power assisted propulsion of cargo ships and hybrid airships\(^4\).
Figure 4. Modeling of SkySails Kite

Figure 5. Lighter than Air Vehicles
This paper concentrates upon another lighter than air application, the pumpkin balloons under development by the NASA Balloon Program Office for its Super-Pressure Balloon Program.

B. Pumpkin Balloon Modeling

The relative storage and computational efficiency of DR as used in inTENS makes it practical to analyze complete balloon models built from several hundred thousand nodes and elements on standard PC hardware. The models themselves comprise constant strain triangular elements for the balloon film, line tension elements to represent the tendons and an assemblage of struts to model the stiff capping plates at base and apex.

Given the size and symmetry of the structures an automated data generator was written to efficiently produce the standard inTENS data files. These generated an initially spherical balloon shape to which inflation pressure was applied, together with specified stresses in the membrane, to establish a preliminary equilibrated pumpkin shape with the correct tendon lengths and an approximate profile of the lobed film between those tendons.

In practice each gore of the balloon between the tendons is fabricated from a single flat piece of polyethylene film. In order to correctly model the as-built structure, this flat pattern geometry is then mapped onto the preliminary mode (Fig. 6) which must then be reanalyzed to establish a correct equilibrium stressed shape.

![Figure 6. Mapping Fabricated Pattern Geometry onto Numerical Model](image)

To fully represent a flight balloon, additional features must be added to the model. The cap, an additional thinner layer of film on the upper portion of the balloon to resist launch-induced stresses, is represented by factoring of local stiffnesses (Fig. 7).

The end fittings are represented by stiff assemblies of strut elements and appropriate selfweights are applied, along with the payload and the buoyancy effects of the enclosed gas. The load tape tendons are modeled as linear
elastic tension elements with an initial strain, according to load/strain data generated by the fabricator, Aerostar, as part of the development of tendon pre-processing procedures.

The most significant analytic difference between a flight balloon and its counterpart ground model is the influence of temperature. Significant additional stresses in the meridional direction are generated because of the differential between the coefficients of thermal expansion (CTE) between the film and the PBO tendons. The latter exhibit effectively no thermal strain, in comparison to the film that has differing values in machine and transverse direction, and values that themselves vary with temperature.

Figure 7. Modeling the Additional Cap Layer

C. Overall Geometric Instability

This analysis effort was originally conceived in order to numerically replicate the problem of overall geometric instability as first investigated by Calladine\(^\text{6}\) in response to problems identified by Nott\(^\text{7}\). This class of instability is identified by the development of a regular wave-shaped distortion from the desired symmetrical fully deployed balloon shape. With increasing pressure it folds inwards on lines of symmetry and low stress. The structure is moving into a shape with greater enclosed volume. This phenomenon is associated with excess lobe material, especially in the polar regions, and uni-axial stressing that prevents control on distortion through the mobilization of shear stiffness. In extreme cases the balloon will not deploy at all, in others it deploys but moves into instability as the pressure is increased.
When present, this effect is replicated in an inTENS simulation by simply increasing the pressure loading on an initially symmetric numerical model of the balloon. Consistent predictions of instability onset have been made by
two independent analysis methods. A 10m diameter test balloon was deliberately designed to never deploy, and the resulting deformed geometry closely matched the numerical prediction.

The possible influences of construction tolerances on balloon performance have also been investigated. Initial analyses of a 0.17mcm flight balloon using theoretical cutting patterns indicated that the system was still geometrically stable at 500Pa, which was twice its design maximum flight pressure. As a first estimate of the possible influences of manufacturing tolerances on stability, a series of studies were made with increasing additional width added to each panel. A constant offset was added to each side of the panel, tapering back to zero over 1.5m lengths at either extremity adjacent to the end fittings. Additions were made progressively until the model showed geometric instability with an additional 20mm in total to each panel lead to instability at 240Pa inflation pressure.

D. Deployment Instability

Flight 555NT of a 6mcf super-pressure balloon in September of 2006 developed a prominent single-cleft distortion during ascent which remained in place on float and through pressurization up to beyond the design maximum pressure of 240Pa.

A subsequent hangar test of the helium inflation and deployment of a 27m diameter scaled model of 555NT has shed new light on the nature of this particular instability. Physical examination of the S-cleft showed that it featured a diagonal band of strong off-meridional stresses as was maintained through inter-surface contact. It was possible to force the s-cleft to migrate from one location to another with the balloon only partially deployed. It appears to be an alternative minimum energy state that can evolve during deployment and from which the balloon is unable to recover as deployment continues and pressure is increased.

![Figure 10. Hangar Test Balloon with Distinctive S-cleft](image)

The S-cleft feature has developed through the influence of helium on the form evolution during ascent, and as a consequence was not seen in any of the previous series of air-filled test balloons. The shape of the balloon whilst partially deployed is complex with significant folding and contact between lobes. Inevitably the maximum height of the lobes varies at the apex of the balloon, and the buoyant helium moves towards the zones of higher elevation, pushing them higher and attracting more gas. The end effect is groups of lobes forming peaks, adjacent to lower folded zones of excess material. As the test proceeded, the group of peaked regions consolidated into one significant peak with the remaining excess film sheared into the adjacent S-cleft.
Physical examination of the S-cleft showed that it featured a diagonal band of strong off-meridional stresses that had developed in a series of adjacent trajectories across the cleft. The magnitude of these stresses was sufficient to make prising apart of the cleft impossible at super-pressure conditions and to keep local hoop stresses to a minimum. The stable equilibrium form with S-cleft is effectively locked in, increasing pressure only serves to reinforce that condition.

Deng and Pellegrino⁹ are using ABAQUS Explicit to investigate deployment stability numerically. The ABAQUS code has the advantage of a powerful general contact algorithm which is essential when trying to model the complexities of this problem.

III. Non Linear Material Modeling

Finite element analyses to date, with linear elastic material model, had been carried out with equivalent elastic properties assessed from an application of the above relations. Typically a single ‘worst case’ stiffness value was applied to a full structure model based, for example, on the maximum anticipated film stress. Although this was an appropriate initial approximation, the ideal desired solution is to integrate the full non-linear material model into inTENS. This would give accurate stress-dependent material properties for each individual finite element in a structure model. By introducing time dependent loading it would allow a proper assessment of the effect of stress relaxation alongside any changes due to variations in temperature and loading.

The non-linear viscoelastic constitutive equations for the linear low-density polyethylene film used for the super-pressure balloons require the modeling of time dependent properties to accurately describe creep and/or relaxation subject to variable thermal and biaxial loading history. Rand et al¹⁰ have previously shown that the material model developed by Schapery¹¹ could be applied to the balloon film. The theoretical development of this model has been presented already, but is summarized here for completeness.

Figure 11. Deployment Modeling (Deng & Pellegrino⁹)
A. Creep Compliance Model

The linear viscoelastic creep compliance $D_\psi$ is defined as the strain response for a unit stress. The current compliance at any time comprises an elastic component $D_0$, which is independent of stress and temperature, and a transient component $\Delta D(\psi)$ where $\psi$ is the reduced time that incorporates the effects of stress and temperature:

$$D_\psi = D_0 + \Delta D(\psi)$$  \hspace{1cm} (5)

The transient component may be expressed as a series of exponentials in reduced time:

$$\Delta D(\psi) = \sum_{n=1}^{N} A_n (1 - e^{-\lambda_r \psi}) \quad \text{where} \quad \lambda_r = 10^{a-b_r}$$  \hspace{1cm} (6)

For biaxial compliance it is has been shown that the transient compliance in any direction may be expressed in terms of a constant multiplied by the compliance in the machine direction:

$$\Delta D_{ij} = S_{ij} \Delta D$$  \hspace{1cm} (7)

The three stresses due to constant biaxial stress may be expressed:

$$\sigma_1 = E_0 (\varepsilon_1 + K_{12} \varepsilon_2) + h_2 \Delta E(\psi)(\varepsilon_1 + K_{12} \varepsilon_2)$$

$$\sigma_2 = E_0 (K_{21} \varepsilon_1 + K_{22} \varepsilon_2) + h_2 \Delta E(\psi)(K_{21} \varepsilon_1 + K_{22} \varepsilon_2)$$

$$\sigma_6 = E_0 K_{66} \varepsilon_6 + h_2 \Delta E(\psi)K_{66} \varepsilon_6$$  \hspace{1cm} (8)

where $g_2$ is a function of stress for the non-linear viscoelastic realm

B. Relaxation Model

The creep compliance approach expresses current material strains in terms of stress through a combination of current incremental strain and preceding strain history. The DR finite element formulation requires an evaluation of stress in terms of strain, such that an iterative solution is required at each analysis step. There may be thousands of steps within each DR equilibrium solution that forms one step of the overall real time stepping sequence, and the resulting additional computation was unrealistic. With that in mind, Rand\cite{11} has re-evaluated the viscoelastic relations in terms of the relaxation modulus.

Finite element analyses are typically calculating element stress in terms of strain calculated from nodal displacements. For such an application it is convenient to recast the viscoelastic formulation in terms of the relaxation modulus rather than the creep compliance.

The current relaxation modulus at any time comprises an elastic portion $E_0$, which is independent of strain and temperature, and a transient component $\Delta E(\psi)$ where $\psi$ is the reduced time that incorporates the effects of strain and temperature:

$$E_\psi = E_0 + \Delta E(\psi)$$  \hspace{1cm} (9)

The transient component may be expressed as a series of exponentials in reduced time:

$$\Delta E(\psi) = \sum_{r=1}^{N} B_r (1 - e^{-\lambda_r \psi}) \quad \lambda_r = 10^{c-d_r}$$  \hspace{1cm} (10)

The three stresses due to constant biaxial strain may be expressed:

$$\sigma_1 = E_0 (\varepsilon_1 + K_{12} \varepsilon_2) + h_2 \Delta E(\psi)(\varepsilon_1 + K_{12} \varepsilon_2)$$

$$\sigma_2 = E_0 (K_{21} \varepsilon_1 + K_{22} \varepsilon_2) + h_2 \Delta E(\psi)(K_{21} \varepsilon_1 + K_{22} \varepsilon_2)$$

$$\sigma_6 = E_0 K_{66} \varepsilon_6 + h_2 \Delta E(\psi)K_{66} \varepsilon_6$$  \hspace{1cm} (11)

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where $K_{ij}$ relates the transient modulus in any direction to that in the machine direction $\Delta E_{ij} = K_{ij}\Delta E$ and $h_2$ is a function of strain for the non-linear viscoelastic realm.

## C. Incremental Implementation

Rand\textsuperscript{12} has expressed the Schapery model into an incremental format, which permits transient analysis in a series of discrete time-steps. The function that carries the entire history necessary for the evaluation of transient strain at a particular time step is dependent only upon information available from the previous time step. This enables a computationally efficient implementation into FE software.

In general the strain and temperature histories will vary over the duration of a flight and an integral approach is needed to assess the current stress $\sigma_1^t$ at elapsed time $t$. This is comprised of elastic and transient components:

\begin{equation}
\sigma_1^t = \sigma_1^0 + \Delta \sigma_1^t
\end{equation}

\begin{equation}
\sigma_1(t) = E_0(\varepsilon_1 + K_{12}^0h_2\varepsilon_2) + \int_0^t \Delta E(t - \tau) \frac{\partial (h_2(\varepsilon_1 + K_{12}^0h_2\varepsilon_2))}{\partial \varepsilon_2} d\tau
\end{equation}

Let $\Theta_1 = h_2(\varepsilon_1 + K_{12}^0\varepsilon_2)$ which varies over the loading history.

Assuming the stress function $\Theta$ occurs in discreet steps 1 to $m$, it can be shown that:

\begin{equation}
\{\Delta E, d\Theta\} = \Delta \sigma^t = \int_0^\psi \Delta E(\psi - \psi') \frac{\partial \Theta}{\partial \psi'} d\psi' = \sum_{r=1}^N B_r [\Theta(\psi_m) - G(r, m)]
\end{equation}

\begin{equation}
G(r, m) = \sum_{j=1}^m \Delta \Theta(\psi_j) e^{-\lambda_r(\psi_m - \psi_{j-1})}
\end{equation}

The entire loading history up to time $\psi_m$ is contained in the function $G(r, m)$ which may be updated for the next time step:

\begin{equation}
G(r, m + 1) = [G(r, m) + \Delta \Theta(\psi_{m+1})]e^{-\lambda_r(\psi_{m+1} - \psi_m)}
\end{equation}

This incremental formulation of the transient behavior of a viscoelastic material is numerically efficient in that information from a single preceding time step is all that is required to compute the change in strain for the next time step.

An alternative implementation by Gerngross\textsuperscript{13} assumes that the stress function $\Theta$ varies linearly over the current time interval. Comparison for a number of test problems has shown no significant differences between the two approaches.

## D. Inclusion in the inTENS Finite Element Analysis Program

The Incremental Schapery Rand formulation permits transient analysis in a series of discrete time-steps for which the function that carries the entire history necessary for the evaluation of transient strain at a particular time step is dependent only upon information available from the previous time step. By using the relaxation modulus approach membrane element stresses may be obtained directly from element strains. This obviates the need for repeated iterations within the Dynamic Relaxation solution process and improves solution times dramatically when compared to the previous creep compliance implementation.

In its inTENS implementation, this Incremental Schapery Rand (ISR) approach requires an incremental sequence of individual Dynamic Relaxation analyses. By storing the element history function at every, or selected, time point, it is possible to restart an analysis sequence from a chosen point rather than needing to return to the beginning. This will be especially appropriate when tracing long time sequences as might be anticipated with the modeling of stress and geometry changes over a balloon flight.
IV. Model Verification

A. Material Model Verification

A large number of laboratory tests have been undertaken to both provide data for the generation of appropriate coefficients for the viscoelastic models. The current relaxation coefficients have been determined by Rand\textsuperscript{12} from numerical experiments undertaken using the creep compliance approach and recent experimental data.

A series of cylinder tests has been undertaken by the Balloon Research and Development Laboratory (BRDL) at NASA’s Wallops Flight Facility. These have simulated a variety of biaxial loading conditions at various temperatures, and are intended to facilitate verification of the material model. Varying biaxial stress ratios can be generated in the cylinder axial and hoop directions by the introduction of inflation pressure and additional axial loading. Film strains are measured by photogrammetry.

Current results are showing reasonable correlation between the two differing material models and experimental results at lower temperature, such as the 2:1 stress ratio case at 253K as shown in figure 12.

The consistency of correlation at higher temperatures is less good, both between the material models and with the test results. Work is ongoing on these models, with investigations into the inclusion of a plastic component to improve the representation of large strains and permanent deformation. Recent analyses have also highlighted the influence of the rate of temperature change on the relaxation model.

Figure 11. BRDL Test Cylinder with Photogrammetry Equipment

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14
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Figure 12. Incremental Analysis of Cylinder Test

B. FE Model Verification

Having verified the basic material model, the next stage is to provide verification of the finite element model of a complete balloon with foil, tendons and end fittings. The interaction between tendons and film is critical to the performance of the balloon. The distinctive pumpkin shape of the balloon reduces film hoop stress by transferring loads into the tendons. The continuous fixity through adhesive of the tendon within its sleeve prevents the migration of film towards the end fittings, which would greatly reduce the geometric stability of the system through excess local material and the inability to mobilize biaxial stress. The tendons also provide meridional strain arrest to the film which limits runaway creep in the presence of high strains due to differential thermal performance.
A 48 gore 8.5m diameter test balloon was fabricated with initially flat facets resulting in a bulge angle of approximately 40deg at its 350Pa design pressure (Fig.13). The balloon was comprehensively targeted for photogrammetric measurement.

The results of an incremental analysis of one of the pressurization cycles applied to this balloon are shown in figure. The meridional strains are generally underestimated, whilst the hoop strains are overestimated with increasing pressure. This could be due to a number of factors. The viscoelastic creep is in the non-linear region due to the relatively high 290K temperature. There is also the possibility of some plastic deformation from a previous stage of the test. Both of these issues should be addressed by currently ongoing work on the material model. The analysis was carried out with 81 steps, each being a full equilibrium solution at the specific time.
V. Flight Simulation by Incremental Analysis

Simulation of a balloon’s structural behavior over the duration of a flight is a key goal of this analysis effort. An understanding of film stresses and balloon shape over anticipated flights of up to 100 days is an essential part of the viability assessment for such missions. As discussed earlier, there are two contrasting requirements that need to be balanced in the design. Excess film material must be minimized to ensure both deployment and overall geometric stability, and the latter must be maintained throughout the flight. The reduction of excess material is achieved by flattening the lobe shape, which will in turn lead to higher stresses and the possibility of increased creep back to an undesirable shape.

The principle variables influencing the balloon during an extended flight, temperature and pressure, are governed primarily by the diurnal cycle with additional influence from climatic and ground conditions. For the NASA Super Pressure Balloon Program the performance profile of a mission is determined using their BalloonAscent software written by Rodger Farley at GSFC. Traces of film temperature and differential pressure against flight time are some of the output options available that cover a large number of variables. BalloonAscent enables the ‘picking’ of selected data points which are stored in a data file. This data is read into PRETENS, a pre-processing component of the inTENS software suite, and the pressure and temperature profiles are recreated using parametric cubic spline interpolation. The time points to be used in the incremental analysis sequence are typically chosen to be closer together when there are significant changes in the temperature and pressure variables.

An example is given in figure 15 of the simulation of a 2mcf flight from Fort Sumner NM. The screen dumps show the trace output from BalloonAscent. The other plots show the variation of stress and strain at the gore centre point over the duration of the 50 hour simulation. This first simulation employed 163 equilibrium solution points with varying time intervals in between. The analysis points can be visualized by the ‘dots’ on the graph traces of temperature and pressure.

Figure 15. 2mcf Balloon Flight Simulation
The simulations were then repeated, firstly with 102 iterations with approximately 30min time intervals and then again with 27 iterations and an approximately 2hr time interval. The results are overlaid on the original plot in figure 16. It can be seen that there is little significant change as the number of iterations has been reduced.

Figure 16. 2mcf Balloon Flight Simulation: Reduced Iterations

The results plotted above relate to a single point on the balloon surface. In order to visualize the stresses and strains for the full gore, complete stress, strain and geometry is stored at each solution point in a Tecplot file format to aid visualization. A video file of the simulation may be subsequently generated in Tecplot, and this gives the readiest access to the large amounts of data involved (Fig. 17). In order to limit retention of excessive amounts of data, standard inTENS output files are created at selected time points only. These include the current viscoelastic history parameters, and enable a new simulation to be continued from one of these select time points rather than returning to time zero. This enables both the breaking down of very long simulations into discrete computational packages and the option to efficiently consider alternative flight profiles at any stage of an operation.

Figure 18 shows the results of the simulation of a forthcoming 14.85mcf super-pressure balloon test flight from Sweden to Northern Canada this June. The significant drop in temperature at the 45 hour time point is representative of a cold storm over Greenland. Again the flight performance predictions were taken from BalloonAscent. At the same time a simulation was run for a possible flight from Vandenberg Air Force Base towards Japan. The results were very counter-intuitive, and subsequent investigation has shown that the relaxation material model is missing a term related to rate of change of temperature. The VAFB flight profile was marked by very much greater rates of temperature change than for the Sweden simulation. Dr Rand is currently updating the material model accordingly, with supporting material tests being undertaken at the BRDL.
Figure 17. Incremental Analysis Stress and Strain Visualization

Figure 18. Sweden Flight Simulation
VI. Stability During Flight

It is necessary to check that changes to the shape of a balloon due to viscoelastic creep over the duration of a flight has not increased its vulnerability to overall geometric instability. Geometric stability has been studied previously by progressively increasing the pressure of an elastic balloon model in an attempt to enforce its onset. Numerical rounding differences in the symmetric FE model are sufficient to seed the onset of instability which develops as the explicit analysis proceeds. In some cases an initial perturbation is applied in the form of an overall sinusoidal up-down distortion. A stable form snaps back to the symmetric pumpkin shape.

The time-stepping viscoelastic simulation of a balloon flight has initially been undertaken using a single lobe of the balloon for reasons of computational economy. The current stress and material total modulus at a particular point in time are likely to be different for each membrane finite element of that model. A geometric stability investigation of the balloon at that particular point in its flight involves the conversion of the viscoelastic model to its elastic equivalent. The equivalent elastic, tangent, stiffness and unstressed geometry are determined for each individual element such that when the elastic model is subject to the same pressure and temperature conditions then its calculated stresses and deformed geometry are identical to that of the analysis.

Referring back to the basic viscoelastic relations:

$$ E_\psi = E_0 + h_2 \Delta E(\psi) $$

where

- $E_\psi$ = element total modulus at time $t$, reduced time $\psi$
- $E_0$ = elastic modulus
- $\Delta E(\psi)$ = transient modulus at time $t$
- $h_2$ is a function of temperature and strain

$$ \sigma = D \varepsilon' $$

where

- $\sigma$ = element stress at time $t$, as determined by non-linear viscoelastic analysis
- $D$ = equivalent elastic material matrix at time $t$
- $\varepsilon'$ = equivalent elastic strains that would generate current element stress $\sigma$ when applied to the material matrix $D$

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix}
= 
(E_0 + h_2 \Delta E(\psi))
\begin{bmatrix}
K_{11} & K_{12} & 0 \\
K_{21} & K_{22} & 0 \\
0 & 0 & K_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon'_1 \\
\varepsilon'_2 \\
\varepsilon'_6
\end{bmatrix}
= 
\begin{bmatrix}
d_{11} & d_{12} & 0 \\
d_{21} & d_{22} & 0 \\
0 & 0 & d_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon'_1 \\
\varepsilon'_2 \\
\varepsilon'_6
\end{bmatrix}
$$

$$
\begin{bmatrix}
\varepsilon'_1 \\
\varepsilon'_2 \\
\varepsilon'_6
\end{bmatrix}
= 
\begin{bmatrix}
\frac{d_{22}}{\alpha} & -\frac{d_{12}}{\alpha} & 0 \\
-\frac{d_{21}}{\alpha} & \frac{d_{11}}{\alpha} & 0 \\
0 & 0 & \frac{1}{d_{66}}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix}
$$

where $\alpha = (d_{11}d_{22} - d_{12}d_{21})$

$\varepsilon'$ is the total equivalent elastic strain determined from the current total stress $\sigma$ at time $t$ and temperature $T$, with components:

$$
\varepsilon' = \varepsilon'_m + \varepsilon'_T
$$

where $\varepsilon'_m$ is the mechanical strain and $\varepsilon'_T$ the thermal strain

In inTENS elastic analyses the unstressed geometry at a reference temperature $T_0$ is retained for each element. Because of the explicit analysis procedure it is possible to use element natural stiffness relations, and for constant strain triangular membrane elements the displacements are defined by the 3 side extensions $\delta$ relative to the unstressed side lengths $l_0$. Rigid body motion is handled by the explicit integration always working in current geometry.
In general element strains at a time $t$ are given by:

$$\epsilon = G \delta$$

where $\delta = l_t - l_0$ and $l_t$ are the element current side lengths and $G$ is a function of current and initial element geometry.

Thus calculating the element unstressed reference geometry $l_0$:

$$\delta_m = G^{-1} \epsilon_m'$$

$$l_0 = l_t - \delta_m$$

The information to be passed from the viscoelastic analysis to the equivalent elastic stability analysis for each element is the material matrix $D$, the unstressed reference geometry $l_0$ and the thermal strains $\epsilon_T$.

When the equivalent elastic model is loaded with the same pressure and temperature distribution as the viscoelastic ‘snapshot’ then the equilibrium geometry, element stresses and element stiffnesses will be identical to those at that ‘snapshot’. This is the only loaded state at which the two models will agree.

Exploiting symmetry, the viscoelastic analysis of a flight profile has been undertaken for a single lobe of the balloon only. In order to undertake the overall instability analysis the single lobe elastic equivalent model is then used to generate a full structure model.

The ‘snapshot’ inflation pressure $P_t$ is then gradually increased in a succession of elastic analyses until the onset of overall geometric instability is determined at $P_{\text{inst}}$.

The ratio $\left(\frac{P_{\text{inst}}}{P_t}\right)$ gives a measure of the vulnerability of the balloon to overall geometric instability at a particular time in its flight profile. This assessment takes account of the influence of geometry change due to creep and of material stiffness due to time, temperature and loading history.

The influence of fabrication tolerances may be considered by running a number of viscoelastic simulations with differing lobe initial geometries. The elasticated element properties of these varying gores may be seeded into the full structure model with varying frequencies to better represent an as-built model when considering geometric instability.

VII. Summary and Conclusions

The NASA Super-Pressure Balloon Program has achieved its first long duration test flight with a record breaking 54 day flight over Antarctica in January 2009. This is the culmination of many years of development and testing. One aspect of the development is the implementation and verification of numerical analysis and design tools in support of the program.

Tensys have a long established membrane structure analysis program suite, inTENS, which is based upon an explicit Dynamic Relaxation solution procedure. This has a natural approach to the large deformation analysis of stressed membrane structures and has been shown capable of modeling the overall geometric instability problems that characterized early super-pressure balloon design.

The polyethylene film used in balloon construction brings its own challenges alongside its light weight and UV resistance. A non-linear viscoelastic material model is necessary to characterize the film. This is subject to ongoing development within the design team. The basic model constituents are summarized in the paper, with the relaxation approach implemented into inTENS in an incremental form. This permits a time-stepping incremental simulation of a flight profile. Work is continuing on extending the material models to account for a plastic behavior component and also the influence of rate of temperature change.

The need for such a numerical model is highlighted by the design requirement for minimal lobing in order to improve stability. The design is walking a narrow path between excessive material stresses and the onset of S-cleft instability during deployment, and reliable analysis is needed to judge the stress relaxation afforded by viscoelasticity against the associated geometry change due to creep.

Examples of the application of the method to pumpkin balloon flight profiles are presented. A procedure is also presented for the assessment of balloon geometric stability at any chosen time point. The stresses, strained geometry and current relaxation modulus are available for each membrane element of a single lobe model at that time. These are mapped onto a full balloon model for study of vulnerability to overall geometric instability with increasing pressure.
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