Active Noise Control inside the Loadmaster Area of a Turboprop Transport Aircraft

Kay Kochan\(^1\) and Delf Sachau\(^2\)

*Helmut-Schmidt-University / University of the Federal Armed Forces Hamburg, Germany*

and

Harald Breitbach\(^3\)

*Airbus Deutschland, Hamburg, Germany*

The paper describes the design and test of a prototype active noise controller for a semi-enclosed cabin inside a turbo-prop aircraft. Here, the eight active noise control loudspeakers and sixteen microphones are mounted at the cabin walls and ceiling. For the development process, an acoustical mock-up is built to reproduce the acoustic situation inside the aircraft on the ground. The acoustic requirements on such a mock-up are described and well-founded by a coupling analysis. Moreover, the controller is based on a frequency domain implementation of the steepest descent algorithm. To achieve the optimal performance and robustness requirements, a method to adjust the microphone and loudspeaker weights is presented. The new method also considers the uncertainty of the system in the controller parameter design process. Experimental results show the efficiency of the prototype system for a typical load case.

**Nomenclature**

\[
\begin{align*}
A_s & = \text{equivalent absorbing surface of a room} \\
B_s & = \text{damping matrix} \\
C & = \text{controller transfer function matrix in the steady state} \\
c_s & = \text{speed of sound in air} \\
d & = \text{column matrix of the primary noise signal at the control microphones} \\
d_M & = \text{column matrix of the primary noise signal at the monitor microphones} \\
E_s & = \text{acoustical energy density inside a room} \\
e_s, e & = \text{error signal and the column matrix of the error signal at the control microphones} \\
e_M & = \text{column matrix of the error signal at the monitor microphones} \\
G & = \text{transfer function matrix between the loudspeakers and error microphones} \\
G_M & = \text{transfer function matrix between the loudspeakers and monitor microphones} \\
i & = \text{general running index} \\
J_{IR} & = \text{controller cost function} \\
K_s & = \text{stiffness matrix} \\
k_s & = \text{coupling factor} \\
M_s & = \text{mass matrix} \\
n & = \text{discrete time} \\
P_{ss} & = \text{acoustic energy}
\end{align*}
\]

\(^1\) Scientific Assistant, Department of Mechatronics, Holstenhofweg 85, D-22043 Hamburg Germany.
\(^2\) Chief of the Mechatronic Department, Holstenhofweg 85, D-22043 Hamburg Germany.
\(^3\) Program Coordinator A400M, Cabin Acoustics, Kreetslag 10, D-21229 Hamburg, Germany.
\( p, \dot{p} \) = pressure and column matrix of the pressure at the nodes of the FEM mesh \\
\( Q \) = controller weighting matrix for the error signals \\
\( R \) = controller weighting matrix for the loudspeaker actuation signals \\
\( S_{12} \) = coupling surface between the loadmaster area and the cargo hold \\
\( t \) = continuous time \\
\( u, u_{\text{opt}} \) = loudspeaker actuation signal and loudspeaker actuation signal in the steady state \\
\( u_{\text{max}} \) = maximum allowed loudspeaker actuation signal \\
\( V_x \) = volume of the cargo hold or loadmaster area \\
\( \hat{s} \) = modal eigenvector \\
\( x, y, z \) = Cartesian coordinates \\
\( \alpha \) = damping factor \\
\( \Delta \) = prefix to indicate the uncertainty of the according value \\
\( \Phi \) = modal matrix with modal eigenvectors \\
\( \rho_0 \) = density of air \\
\( \omega_0 \) = eigenfrequencies \\
\( 0 \) = index to indicate the nominal transfer function or primary noise field \\
\( \| \cdot \|_F, \| \cdot \|_2 \) = Frobenius norm and 2-norm

I. Introduction

The interior sound field in propeller-driven aircrafts are mainly affected by high tonal sound pressure levels. This is caused by the propeller blades which pass the fuselage at certain angular frequencies (BPF, blade passing frequency), see Fig. 1. The vibrating fuselage structure excites the interior air volume. Inside the cabin, the sound pressure level may be higher than 100 dB(A) without noise treatments. The designer of an aircraft is obliged to assemble passive noise insulation material and/or to design an active noise control system to comply with the noise specifications and regulations, see for example the European Directive 2003/10EC\(^1\). The active noise control method is superior for low frequency noise, when compared to passive noise insulation methods\(^2\). Such an active noise control (ANC) system consists of loudspeakers and microphones connected to a controller which calculates loudspeaker actuation signals \( u \) to minimize the disturbing sound pressure \( d \) at these microphones, see Fig. 2.

---

Figure 1. Transport Aircraft  
Figure 2. Active Noise Principle

American Institute of Aeronautics and Astronautics
In this paper, the active noise control method is employed to the acoustical problem of the loadmaster area, see Figure 1. Eight loudspeakers and sixteen microphones are installed on the ceiling and the exterior wall of the loadmaster area. The loadmaster area is a small semi-enclosed volume connected to the large cargo hold. This coupling of a small compartment (loadmaster area) with a large compartment (cargo hold) leads to new questions for the active noise control system design: How can the coupling be simulated in a laboratory environment? Which influence does the coupling have on active system requirements?

Moreover, in the final design process, the hardware and positions of the active noise control loudspeakers and error microphones are already defined. But the controller software and parameters can still be adapted in order to improve the performance and robustness of the system. Questions that are more closely related to industrial applications are important here, such as: Which parameter setup rules can be used to guarantee robust controller behaviour? How should the controller parameters be set to achieve the necessary controller performance and stability?

This paper is organized as follows: For the acoustic ground test, the acoustic coupling between the loadmaster area and the cargo hold is examined for a simplified model using two theoretic methods. The first deploys an energy based method which is focused on the acoustical energy flow in such a coupled system. The second uses a substructure technique where the shapes of the sound fields inside a small loadmaster area are in focus. Based on that analysis, an acoustic ground test facility is developed. Here, the room acoustic considerations and the prototype active noise controller are in the center of the chapter. Afterwards, a controller parameter design method is introduced which involves independent loudspeaker and microphone weights in order to achieve the necessary active noise control performance. At the end, some experimental active noise control results are presented.

II. Coupling Analysis of the Loadmaster Area and the Cargo Hold

As mentioned above, the special characteristic of the supposed turboprop aircraft is that the relatively small volume of the loadmaster area is coupled to a large cargo hold. To identify the basic physical behavior of such a coupled system and to define requirements for an acoustic ground test, a simplified model of an enclosed sound field is considered.

A. Modeling of the coupled System

As illustrated in Figure 3, the cargo hold is represented by a rectangular compartment (volume $V_1 \approx 405 \text{ m}^3$). In addition, the loadmaster area with a volume $V_2$ of approximately $4.5 \text{ m}^3$ is attached at the front of the cargo hold. The coupling surface $S_{12}$ between both compartments is approximately $2.6 \text{ m}^2$ large. The loadmaster area has a slightly curved outer wall which is implied by the chamfer. The reverberation time inside the volumes is assumed to be comparable to the real aircraft ($T_{60} = 0.68$).

![Figure 3. Simplified model of the coupled system loadmaster area and cargo hold](image)

Tonal excitation ($f < 300 \text{ Hz}$) of the sound field inside the aircraft is assumed to be primarily through the fuselage structure in the propeller plane. Due to the synchrophasing of the aircraft propellers, the noise entry into the aircraft is in phase on both sides. Inside the aircraft, the sound waves are partly reflected and absorbed at the interior walls. Hereby the major dissipation effect is induced by the absorption on the boundaries, in comparison to
the minor effect of internal friction in the fluid. The primary noise field in the loadmaster area is excited via the 
coupling surface \(S_{12}\). If the active noise control system is switched on, the primary noise field inside the loadmaster 
area is superimposed by a secondary noise field. This secondary noise field is generated by the active noise control 
loudspeakers at the exterior wall and the ceiling of the loadmaster area.

In the next two sections, we can use the simplified model twice. During the application of the energy based 
method, only the compartment volumes and the equivalent absorbing surfaces of the model are of interest. In the 
substructure based method, the simplified geometry is only evaluated in two dimensions. This strong simplification 
of the real aircraft is tolerable, due to the fact that only the principal physical behavior and tendencies are of interest.

B. Acoustical Energy Flow in the Coupling Surface

First of all, the energy flow in the simplified coupled system is of great interest. Here, the volumes and absorbing 
surfaces as shown in Figure 3 are considered. In the theory of room acoustics, the law of conservation of the 
acoustical energy can be applied to enclosed sound fields\(^4\). The overall acoustical energy of is considered as 
constant. At the system boundaries, acoustical energy can be dissipated at absorbing surfaces or radiated by 
acoustical sources. From this point of view, the acoustical power balance for the cargo (index 1) and the loadmaster 
area (index 2) can be written as

\[
\begin{align*}
E_1 - \frac{c_s}{4} A_{10} E_1 - \frac{c_s}{4} S_{12} E_1 + \frac{c_s}{4} S_{12} E_2 &= 0 \\
E_2 - \frac{c_s}{4} A_{20} E_2 + \frac{c_s}{4} S_{12} E_1 - \frac{c_s}{4} S_{12} E_2 &= 0.
\end{align*}
\]

The variables \(E_1\) and \(E_2\) denote the acoustical energy density in the cargo hold and the loadmaster area. The 
absorbing surfaces in both compartments can be summarized to the equivalent absorbing surfaces \(A_{10}\) and \(A_{20}\) 
respectively. As indicated above, the surface which separates the loadmaster area from the cargo hold is described as 
\(S_{12}\). At this surface, both compartments are able to exchange acoustical energy. Furthermore, \(P = P_1 + P_{12}\) and \(P_2\) 
denotes the acoustical power which is radiated by the acoustic sources in the according compartments. The 
abbreviation \(c_s\) denotes the speed of sound. If we further introduce the abbreviations \(A_{11} = A_{10} + S_{12}\) and 
\(A_{22} = A_{20} + S_{12}\), equation (1) can be rewritten as

\[
\begin{align*}
P_1 - A_{11} E_1 + S_{12} E_2 &= 0 \\
P_2 - A_{22} E_2 + S_{12} E_1 &= 0.
\end{align*}
\]

In the further analysis, two cases will be studied. The first one addresses the case where the primary noise passes 
through the fuselage and excites the cargo hold. The second one deals with the excitation of an active noise control 
loudspeaker which is placed in the loadmaster area. The other corresponding compartment then is only excited via 
the coupling surface \(S_{12}\).

1. Excitation of the Primary Noise Field

During the primary excitation, the acoustical power of the system is introduced via the primary source \(P_1\) 
\((P_2 = 0)\). The energy density \(E_1\) can be evaluated using the equation system (2):

\[
E_1 = \frac{4}{c_s} \frac{P}{A_{11} + \frac{S_{12}}{A_{22}}} = \frac{4}{c_s} \frac{P}{A_{10} + k_1 A_{20}}.
\]

The denominator shows that the contribution of the acoustical absorbing surface \(A_{20}\) to the energy density \(E_1\) is 
reduced by the coupling factor \(k_1 = S_{12}/A_{22}\). If the limit of a sound soft loadmaster area is considered, the second 
summand in the denominator of Eq. (3) changes to:
\[
\lim_{A_{20} \to \infty} \frac{S_{12}}{A_{20} + S_{12}} A_{20} = S_{12}.
\] (4)

On the other hand, the limit of the coupling factor \( k \) for a sound hard loadmaster area leads to

\[
\lim_{A_{20} \to \infty} \frac{S_{12}}{A_{20} + S_{12}} = 1.
\] (5)

Insertion of the limits in the equation (3) leads to the energy density expression for the limit of a sound soft loadmaster area \( E^{ss}_1 \)

\[
E^{ss}_1 = \frac{4}{c_s} \frac{P}{A_{10} + S_{12}}
\] (6)
and the energy density expression for the limit of a sound hard loadmaster area \( E^{sh}_1 \)

\[
E^{sh}_1 = \frac{4}{c_s} \frac{P}{A_{10} + A_{20}}
\] (7)

Equation (6) shows that the coupling surface can be seen as a full absorbing surface for the limit of a sound soft loadmaster area. Furthermore, for a sound hard loadmaster area the coupling is large.

The energy density in the loadmaster area can be obtained by

\[
\frac{E_2}{E_1} = \frac{S_{12}}{A_{22}} = k_1.
\] (8)

If the coupling factor \( k_1 \) is close to 1, the acoustical energy is uniformly distributed in both compartments. On the other hand, if the coupling factor \( k_1 \) is small, a significant difference between the energy density \( E_1 \) and \( E_2 \) can be expected.

2. Actuation of the Noise Control Loudspeakers

In the case of an actuation with the noise control loudspeaker, the acoustical power of the system is introduced via the secondary source \( P_2 \) \( (P_1 = 0) \). The energy density \( E_2 \) can again be evaluated using the equation system (2):

\[
E_2 = \frac{4}{c_s} \frac{S_{12}}{A_{10} + A_{20}} A_{11} = \frac{4}{c_s} \frac{P}{k_2 A_{10} + A_{20}}.
\] (9)

Corresponding to the analysis above, the denominator shows that the contribution of the acoustical absorbing surface \( A_{10} \) to the energy density \( E_2 \) is now reduced by the coupling factor \( k_2 = S_{12} / A_{11} \). Therefore, the limit of a sound soft cargo hold changes equation (9) to

\[
E^{ss}_2 = \frac{4}{c_s} \frac{P}{S_{12} + A_{20}}
\] (10)
and for a sound hard cargo hold to

\[
E^{sh}_2 = \frac{4}{c_s} \frac{P}{A_{10} + A_{20}}.
\] (11)
As analyzed above, for the sound soft limit the coupling factor $k_2$ is small, the coupling to the cargo hold is low. On the other hand, the coupling will be large for a sound hard cargo hold. The ratio of the energy density can be calculated with the system of equations (2)

$$\frac{E_1}{E_2} = \frac{S_{12}}{A_{11}} = k_2.$$  \hfill (12)

With this equation, the ratio of the energy density can be calculated from the ration of the surfaces $S_{12}$ and $A_{11}$.

C. Modal Behavior inside the Loadmaster Area

In section B, an analysis was done to understand coupling in the sense of the energy flow. The aim of the following analysis is to understand the shape of the sound field in a small compartment (loadmaster area) which is coupled to a large compartment (cargo hold). At low frequencies, the acoustic sound field can be described by the wave theory. Here, the enclosed sound field is composed by a linear combination of independent acoustical modes. The description of the sound field can be developed from a second order homogenous linear partial differential equation which is known as acoustic wave equation:

$$\frac{1}{c_s^2 \rho_0} \frac{\partial^2 p(x,y,z,t)}{\partial t^2} - \frac{1}{\rho_0} \Delta p(x,y,z,t) = 0.$$  \hfill (13)

Here, the variable $p$ refers to the acoustic pressure, the constant $\rho_0$ describes the density of air, and

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$  \hfill (14)

stands for the Laplace-operator of the Cartesian coordinates $x$, $y$ and $z$. The product $c_s^2 \rho_0$ specifies the adiabatic bulk modulus or room stiffness.

The acoustic wave equation (13) can be discretized by the finite element (FE) method:

$$\hat{M} \hat{p} + \hat{K} \hat{p} = 0$$  \hfill (15)

where $M$ represents the mass matrix, $K$ is the stiffness matrix and

$$\hat{p} = [\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_N]$$  \hfill (16)

is the column matrix of the pressure at the $N$ FE-nodes.

In order to solve the homogenous differential equation numerically, the introduction of dimension factors $\bar{m} \left( \bar{M} = \bar{m} \bar{M} \right)$ and $\bar{k} \left( \bar{K} = \bar{k} \bar{K} \right)$ as well as a non-dimensional time $\bar{t} = \omega_k t$ with $\omega_k^2 = \bar{k} / \bar{m}$ is appropriate. This leads to the following differential equation:

$$\ddot{\bar{M}} \bar{p} + \bar{K} \bar{p} = 0$$  \hfill (17)

where the second derivative with respect to the non-dimensional time is abbreviated with the large circles above the variable. To involve damping effects, the homogenous equation (17) can be extended by a damping matrix $B$:

$$\ddot{\bar{M}} \bar{p} + \bar{B} \dot{\bar{p}} + \bar{K} \bar{p} = 0$$  \hfill (18)

which represents the internal friction in the fluid and is supposed to be a small stiffness proportional damping with
\[ \ddot{\mathbf{B}} = \alpha \mathbf{K} \]  

(19)

In the case of the two coupled compartments (cargo hold – index 1, loadmaster area – index 2), the equation (18) of the enclosed sound field can be decomposed:

\[
\begin{bmatrix}
\ddot{\mathbf{M}}_1 & 0 & \ddot{\mathbf{M}}_{1C} \\
0 & \ddot{\mathbf{M}}_2 & \ddot{\mathbf{M}}_{2C} \\
\ddot{\mathbf{M}}_{1C}^T & \ddot{\mathbf{M}}_{2C}^T & \ddot{\mathbf{M}}_C
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{p}}_1 \\
\ddot{\mathbf{p}}_2 \\
\ddot{\mathbf{p}}_C
\end{bmatrix}
+ \begin{bmatrix}
\ddot{\mathbf{B}}_1 & 0 & \ddot{\mathbf{B}}_{1C} \\
0 & \ddot{\mathbf{B}}_2 & \ddot{\mathbf{B}}_{2C} \\
\ddot{\mathbf{B}}_{1C}^T & \ddot{\mathbf{B}}_{2C}^T & \ddot{\mathbf{B}}_C
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{p}}_1 \\
\ddot{\mathbf{p}}_2 \\
\ddot{\mathbf{p}}_C
\end{bmatrix}
+ \begin{bmatrix}
\ddot{\mathbf{K}}_1 & 0 & \ddot{\mathbf{K}}_{1C} \\
0 & \ddot{\mathbf{K}}_2 & \ddot{\mathbf{K}}_{2C} \\
\ddot{\mathbf{K}}_{1C}^T & \ddot{\mathbf{K}}_{2C}^T & \ddot{\mathbf{K}}_C
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{p}}_1 \\
\ddot{\mathbf{p}}_2 \\
\ddot{\mathbf{p}}_C
\end{bmatrix}
= \begin{bmatrix}
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0}
\end{bmatrix}.
\]

(20)

For this, the pressure column vector \( \ddot{\mathbf{p}} \) is resorted to collect the degrees of freedom for the individual substructures \( \ddot{\mathbf{p}}_1 \) and \( \ddot{\mathbf{p}}_2 \) as well as the coupling degrees of freedom \( \ddot{\mathbf{p}}_C \). The mass matrix is decomposed in the corresponding substructure mass matrices \( \ddot{\mathbf{M}}_1 \) and \( \ddot{\mathbf{M}}_2 \) as well as the coupling mass matrix \( \ddot{\mathbf{M}}_C \). The damping matrix \( \ddot{\mathbf{B}} \) and the stiffness matrix \( \ddot{\mathbf{K}} \) are similarly fragmented.

If the coupling degrees of freedom \( \ddot{\mathbf{p}}_C \) are set to zero, the equation (20) can be decomposed into two substructures:

\[
\begin{align*}
\ddot{\mathbf{M}}_1 \ddot{\mathbf{p}}_1 + \ddot{\mathbf{B}}_1 \ddot{\mathbf{p}}_1 + \ddot{\mathbf{K}}_1 \ddot{\mathbf{p}}_1 &= 0 \\
\ddot{\mathbf{M}}_2 \ddot{\mathbf{p}}_2 + \ddot{\mathbf{B}}_2 \ddot{\mathbf{p}}_2 + \ddot{\mathbf{K}}_2 \ddot{\mathbf{p}}_2 &= 0
\end{align*}
\]

(21)

Solving the eigenvalue problem leads to the modal matrices of the coupled system (Eq. (20)):

\[
\mathbf{\Phi} = [\mathbf{\hat{x}}_{01}, \mathbf{\hat{x}}_{02}, \ldots, \mathbf{\hat{x}}_{N0}]
\]

(22)

and of the two substructures (Eq. (21))

\[
\mathbf{\Phi}_1 = [\mathbf{\hat{x}}_{11}, \mathbf{\hat{x}}_{12}, \ldots, \mathbf{\hat{x}}_{N1}]
\quad \mathbf{\Phi}_2 = [\mathbf{\hat{x}}_{21}, \mathbf{\hat{x}}_{22}, \ldots, \mathbf{\hat{x}}_{N2}]
\]

(23)

Here, \( \mathbf{\hat{x}}_1 \), \( \mathbf{\hat{x}}_2 \) and \( \mathbf{\hat{x}}_3 \) are the eigenmodes of the according structures. Following the nomenclature of Craig\(^5\), the substructure modes in \( \mathbf{\Phi}_1 \) and \( \mathbf{\Phi}_2 \) are fixed-interface modes which correspond to two separate compartments with an ideal sound soft surface at \( S_{12} \).

The according complex eigenvalues contain the eigenfrequencies \( (\omega_i = \omega_0 \tilde{\omega}_i) \) of the coupled system:

\[
[\omega_{01}, \omega_{02}, \ldots, \omega_{N0}]
\]

(24)

and the eigenfrequencies of the two substructures

\[
[\omega_{11}, \omega_{12}, \ldots, \omega_{N1}]
\quad [\omega_{21}, \omega_{22}, \ldots, \omega_{N2}].
\]

(25)

In order to explain some principle coupling effects of the small loadmaster area with the cargo hold, the simple two-dimensional model can be used as shown in Figure 3. Therefore, the coupled system of Figure 3 was modelled in two-dimensions with the finite element software \textit{COMSOL\textsuperscript{TM}} version 3.5. Altogether, 1793 linear-Lagrange quadrilateral elements were used in the entire model; of which 43 linear-Lagrange elements were deployed in the substructure of the loadmaster area. This corresponds to at least 5 nodes per wavelength up to 300 Hz. Using the export function of \textit{COMSOL\textsuperscript{TM}}, the mass matrix \( \mathbf{M} \) and stiffness matrix \( \mathbf{K} \) were exported to \textit{MATLAB\textsuperscript{TM}} version 2008a. Afterwards, the damping matrix \( \mathbf{B} \) was calculated using the stiffness matrix, see Eq. (19). Here, the parameter \( \alpha \) was set to 2% to introduce a weak damping. With the mass, damping and stiffness matrix, the mode
shapes of the coupled system and the substructures can be evaluated using the equations (20) to (23) and the MATLAB™ function \texttt{polyeig}.

In Figure 4, the eigenfrequencies for the simplified coupled system (cargo hold + loadmaster area) and the substructure of the loadmaster area are displayed up to 300 Hz. As expected the amount of eigenfrequencies increases according to the room size. While the decoupled small loadmaster area has only six eigenfrequencies up to 300 Hz, the large coupled system has 242 eigenfrequencies.

In order to compare the mode shapes of the substructure of the loadmaster area and the corresponding subsection of the coupled system, the modal assurance criterion (MAC) was applied. In Figure 4, the eigenfrequencies with MAC values higher than 0.75 are connected via lines. The gray scale of the lines varies according to the MAC value (MAC = 0.75 – with line; MAC = 1.0 – black line). We can see that the mode shapes of the small loadmaster area substructure can be found in the mode shapes of the entire coupled system. Hereby, the largest similarity follows a certain pattern. It can be found below and above the substructure eigenfrequencies. The coupling of the lengthwise modes (e.g. \( \omega_{22} \)) is stronger than the transversal modes (e.g. \( \omega_{02} \)). On the other hand, a clear allocation between the mode shapes of the cargo hold substructure and the coupled system could not be observed. This is caused by the considerably higher modal density of the large cargo hold.

![Figure 4. Loadmaster area substructure eigenfrequencies allocated to the eigenfrequencies of the coupled system. Detail of the pressure mode shape inside the loadmaster area.](image)

For a visual comparison, the substructure mode shapes and a selection of the mode shapes of the coupled system are also shown in Figure 4. Only the mode shape subsections of the loadmaster area are displayed for the coupled system. When comparing the mode shapes, the similarity can also be found with only small variations close to the coupling surface. This is caused by the usage of fixed-interfaced modes which has zero pressure at the coupling surface.

American Institute of Aeronautics and Astronautics
D. Consequences

For the supposed aircraft, the equivalent absorbing surface $A_{10}$ of the cargo hold is much larger than the equivalent absorbing surface $A_{20}$ of the loadmaster area. Furthermore, the coupling surface $S_{12}$ is larger than the equivalent absorbing surface $A_{20}$.

The analysis shows that for the primary excitation the coupling factor $k_1 = S_{12}/A_{22}$ is slightly smaller than 1. If equation (8) is evaluated, the energy density in both compartments will be equal. Hence, the coupled system can be seen as one system from the viewpoint of the primary excitation when the considered conditions are applied. This denotes that high sound pressure levels inside the cargo hold will propagate unimpededly in the loadmaster area.

From the viewpoint of an excitation with the ANC loudspeaker, the coupling surface $S_{12}$ can be assumed as a full absorbing surface. The acoustical energy density in the cargo hold will be smaller than inside the loadmaster area. Therefore, despite the sound hard material in the loadmaster area, the loadmaster area can be considered as a highly damped room. This has two consequences for the active noise control system. First, it tends to smooth transfer functions between the ANC loudspeaker and the control microphones with lower resonance peaks and a steady slope of the phase responses. This is advantageous for a robust active noise control system because it reduces the sensitivity in the estimated transfer functions, see Ref. 6. Secondly, due to the higher damping, smaller resonance effects induced by the ANC loudspeakers occur in the semi-enclosure. This leads to the requirement of ANC loudspeakers with high performance.

The analysis of the modal behavior has shown that only a few independent mode shapes influence the acoustic behavior inside the loadmaster area. These mode shapes are defined by the local geometry of the loadmaster area and should be non-sensitive to changes in the cargo hold. The modal density is given by the coupled system and will be high in comparison to a decoupled loadmaster area.

III. Experimental Setup for the Acoustic Ground Test

This chapter describes the acoustic ground test facility to simulate the acoustic situation inside the aircraft. Afterwards, the electro-acoustic components of the experimental setup are listed. Moreover, the active noise controller implemented in the frequency domain will be briefly introduced.

A. Acoustic Ground Test Mock-up

According to the analysis above, an acoustical mock-up can be used for the active noise control experiment. This mock-up should reproduce the significant geometric circumstances depending on the shortest important wave length. The primary noise field is excited mainly through the coupling surface. Therefore, the mock-up can be made of wood; an expensive vibro-acoustic mock-up is not necessary. The cargo hold is simulated by an appropriate laboratory. That laboratory room should have a similar mode density which can be fulfilled if the room is large enough. The primary excitation can be realized with loudspeakers which are able to excite the room modes. The reverberation time should be comparable to the real cargo hold.

An experimental setup which satisfies these requirements is shown in Figure 5. The mock-up of the loadmaster area was constructed according to the digital mock-up and was assembled inside a laboratory room which has similar dimensions to the cargo hold, see Ref. 7. The mock-up geometry was simplified corresponding to manufacturing possibilities and the shortest sound wave length. Afterwards, the mock-up was equipped with a honeycomb lining. The reverberation time inside the laboratory is about 0.6 seconds. At the supposed propeller plane, two public address loudspeakers were placed on both sides of the laboratory to excite the primary noise field.
Figure 5. The wooden mock-up of the loadmaster area (LMA) placed inside a laboratory.

B. Electro-Acoustic Components

In the present active noise control experiment, \( N_L = 8 \) loudspeakers and \( N_E = 16 \) microphones were mounted at the exterior wall and the ceiling of the loadmaster area. The transducer positions of the ANC system were aligned on the results of an optimization study which was performed at a former mock-up, refer to Ref. 8. The active control algorithm was implemented on a rapid prototyping controller board (dSPACE 1103).

Moreover, to evaluate the performance of the active noise control system, two ear microphones of an artificial head (HEAD acoustic HMS III) and 12 microphones (Brüel & Kjær B&K 4188) were arranged to measure the sound pressure level in a monitor volume around the supposed head of a sitting person, see Figure 9. The signals were simultaneously analyzed with a second dSPACE 1103 and a B&K LAN-Xi front-end module. A B&K 2694 16-channel DeltaTron conditioning amplifier was used as power supply for the \( N_M = 14 \) monitor microphones.

C. Active Noise Controller

For tonal active noise control, different control algorithms can be used. For the present experiment, a feedforward frequency domain implementation is chosen. The selection criteria are a robust behavior in the context of uncertainties, a low computational complexity due to the large amount of transducers, the accessibility of a reference signal and simple parameter adjustability. A minor criterion in the present application is the convergence performance due to the fact that frequency shifts of the propeller engines will be slow in comparison to the algorithm time constant. The controller may be able to control a fundamental frequency, the first blade passing frequency (1BPF), and two higher harmonic frequencies (2BPF and 3BPF).

The principle scheme of the controller is shown inside the grey box of Figure 6. The time domain signal of the reference \( x(t) \) and the error \( e(t) \) are fed to the controller. Afterwards, the error signals \( e(t) \) are transformed in the frequency domain using a Fast-Fourier-Transformation (FFT). The reference signal is used to discard the idle bins of the complex frequency spectrum. Here, at each discrete time step \( n \), the complex frequency bins of 1BPF \( i = 1 \), 2BPF \( i = 2 \) and 3BPF \( i = 3 \) are grouped to the according column matrices

\[
e(n) = \begin{bmatrix} e_1^1(n), & \ldots, & e_1^i(n), & \ldots, & e_1^n(n) \end{bmatrix}.
\]

(26)

The complex loudspeaker actuation signals \( u_j \) are calculated in each controller \( C_j \) according to the iterative minimization of the controller cost function \( J_{ir} \)

\[
J_{ir} = e_{ir}^H Q e_{ir} + u_{ir}^H R u_{ir}.
\]

(27)
The weighting matrices $Q_i$ and $R_i$ in equation (27) are diagonal matrices which are used to weight the microphones and loudspeakers independently. These weightings have a similar mathematical effect as different loudspeaker and microphone positions and change the phase and amplitude of the loudspeaker actuation. Therefore, changing the weighting matrices $Q_i$ and $R_i$ also affects the residual noise at the monitor microphones. These $N_E + N_L$ weighting parameters of $Q_i$ and $R_i$ have to be adjusted during the controller design process, see chapter IV. The minimization of the controller cost function is carried out by the iterative update equation

$$u_i(n+1) = [I - 2\mu_i R_i] u_i(n) - 2\mu_i G_i^H Q_i e_i(n)$$

(28)

with the step size $\mu_i$. The resulting complex actuation signals $u_i$ are transformed in the time domain with an inverse Discrete-Fourier-Transformation (iDFT). Moreover, to ensure a safe controller service in the case of algorithm instability, a fail-safe function is included in the controller. This function checks at each iteration the magnitude of the loudspeaker actuation signals and switches the controller off if necessary.

The real-time implementation above was realized with a sample-time of 4096 Hz. A Hamming-window with 2048 coefficients and ¾ overlapping were chosen for the FFT. This leads to a controller update rate of 8 Hz which is adequate for the slow frequency shift of the engines. The computational load of the real-time processor was about 27%.

Figure 6. The experimental active noise controller implemented on the dSPACE 1103.

Figure 7. Frequency domain block diagram of the control system.

IV. Controller Parameter Design Method

The determination of the weighting matrices $Q$ and $R$ is in focus of the controller design method. For each frequency, one set of weighting matrices $Q$ and $R$ has to be defined. Therefore, the following method has to be performed for each frequency. For the remainder of the chapter the explicit dependency on the frequency will be dropped for clarity.

In Figure 7, the block diagram of the control system is shown. The residual sound pressure at the error microphones $e$ are the sum of the primary noise field $d$ and the product of the transfer function matrix $G$ with the actuation signal $u$. Simultaneously, the loudspeaker excitation effects the sound pressure at the $N_M$ monitor microphones via the transfer function matrix $G_M$. This leads to

$$e = Gu + d$$

and

$$e_M = G_M u + d_M.$$

(29)
The primary noise $d$ and $d_m$ are assumed to be excited via the same tonal noise source. Differentiation of the cost function $J_q$ according to $u$ leads to the steady state controller transfer function matrix $C$

$$C = - \left( G_0^H Q + R \right)^{-1} G_0^H Q. \quad (30)$$

The problem of the design method is to affect the behavior of the controller to achieve the lowest residual error at the monitor microphones $e_M$. Hereby, the nominal design is based on the nominal plant. On the other hand, the robust design considers additional uncertainties of the plant.

### A. Nominal Controller Design Method

In the case of the nominal controller design the transfer function matrices as well as the primary noise fields are assumed to be constant at their nominal values. Therefore, we can set

$$G_M = G_{M_0}, \quad G = G_0, \quad d = d_0, \quad d_M = d_{M_0}, \quad \text{and} \quad u_{\text{opt}} = u_{\text{opt0}}. \quad (31)$$

With this nominal plant, the performance of the adaptive controller can be evaluated with the energetic mean residual sound pressure level at the $N_M$ monitor microphones. Hence, the residual mean squared error is defined as the performance criterion which has to be minimized:

$$\min_{Q,R} \quad 10 \log_{10} \left\| e_{M_0} \right\|^2. \quad (32)$$

In the sense of an optimization problem, $Q$ and $R$ are the optimization variables and the residual sound pressure in the monitor volume is the objective function.

Moreover, this optimization has to be performed according to constraints. The control stability of the steepest descent adaptive controller can be guaranteed if the smallest eigenvalue $\lambda_{\text{min}}$ of the matrix $\left[ G_{M_0}^H Q G_0 + R \right]$ is positive. This fact is used as the stability constraint, see Eq. (33) b). Moreover, the loudspeaker actuation should not be larger than the maximum allowed actuation amplitude $u_{\text{max}}^{\text{opt0}}$, which leads to the constraint c) in Eq. (33). Finally, to achieve a convex controller cost function $J_q$, the weighting matrices $Q$ and $R$ has to be positive definite. This leads to the constraints d) and e) in Eq. (33) where $\triangleright$ denotes the positive eigenvalues of the matrices.

Using the Eq. (30), (32) and the constraints, the nominal design problem can be written as an optimization problem:

$$\min_{Q,R} \quad 10 \log_{10} \left\| G_{M_0} \cdot u_{\text{opt0}}(Q,R) + d_{M_0} \right\|^2\quad \text{a)}$$

subject to:\n
$$\min_{i=1}^{N_M} \left\{ \text{eig} \left( G_{M_0}^H Q G_0 + R \right) \right\} > 0, \quad \text{b)}$$

$$\max_{i=1}^{N_M} \left\{ u_{\text{opt0}}(i) \right\} \text{max_{opt0}} > 0, \quad \text{c)}$$

$$Q \triangleright 0, \quad \text{d)}$$

$$R \triangleright 0, \quad \text{e)}$$

$$u_{\text{opt0}}(Q,R) = - \left( G_0^H Q G_0 + R \right)^{-1} G_0^H Q d_0 \quad \text{f).}$$

This nonlinear constrained optimization problem can be solved by using a genetic algorithm. The optimal weighting matrices $Q$ and $R$ are found when the constraints are satisfied and the best control performance is achieved.

### B. Robust Controller Design Method

The simplified mathematical models of the plant above are an abstraction of the dynamic behavior of real time-variant systems with weak nonlinearities. It is assumed that the real physical plant varies around the nominal plant.
The implemented model in the controller equates the nominal model. Hence, according to Equation (31), we set for the physical plant

\[ G_M = G_M^0 + \Delta G_M \quad \text{d} = d_0 + \Delta d \]
\[ G = G_0 + \Delta G \quad \text{d}_M = d_{M0} + \Delta d_M \]
\[ u_{\text{opt}} = u_{\text{opt}0} + \Delta u_{\text{opt}} \tag{34} \]

where the variables labeled with \( \Delta \) represent the additive modeling errors. Here, the modeling errors are arbitrarily and only known in their norm.

Similar to the optimization problem above, a robust optimization problem is defined which combines the robust control stability and robust control quality criterion. The solution of the optimization problem has to guarantee the stability and has to achieve the best performance for all norm-bounded uncertain plants. In other words, the objective function and the constraints should be evaluated for the worst-case stability and worst-case performance. The worst-case can be found by solving a maximization problem according to the variables \( \Delta G_M, \Delta d_M, \Delta G \) and \( \Delta d \). However, such a minimax optimization problem is not solvable in an efficient manner.

On the other hand, the worst-case can be estimated after a longer calculation by algebraic formulas, refer to Ref. 11. For example, the worst-case actuation can be approximated by using the Sherman-Morrison-Woodbury formula\(^{12}\) for an estimation of the variable \( \Delta u_{\text{opt}} \)

\[ \Delta u_{\text{opt}} (Q, R) \approx C\Delta d + C\Delta G u_{\text{opt}0} (Q, R). \tag{35} \]

Moreover, using the triangle inequality and neglecting the quadratic terms leads to an estimation of the residual noise at the monitor microphones:

\[ \max_{\Delta} \left\| (G_{M0} + \Delta G_M) u_{\text{opt}} (Q, R) + d_{M0} + \Delta d_M \right\|_2 \leq \max_{\Delta G_M} \left\| \Delta G_M u_{\text{opt}} \right\|_2 + \max_{\Delta d} \left\| G_M \Delta d \right\|_2 + \max_{\Delta G} \left\| C \Delta G u_{\text{opt}} \right\|_2 \tag{36} \]

Equation (36) can be easily estimated if the norms of \( \Delta G_M, \Delta d_M, \Delta G \) and \( \Delta d \) are known. For example, if the norm \( \left\| \Delta d \right\|_2 \) is known, the third term can be approximated in the following manner:

\[ \max_{\Delta G} \left\| C \Delta d \right\|_2 < \left\| G_M \right\|_2 \left\| \Delta d \right\|_2 \tag{37} \]

The worst-case stability follows by the lower bound of the smallest eigenvalue of the matrix \( [G_0^0 Q(G_0 + \Delta G) + R] \), see [1]. This eigenvalue can be estimated by

\[ \hat{\lambda}_i(Q, R) = \lambda_i(Q, R) - x_i^H G_0^0 Q \Delta G x_i \tag{38} \]

where \( \lambda_i \) are the eigenvalues and \( x_i \) are the eigenvectors of the matrix \( [G_0^0 QG_0 + R] \), see Ref. 13. According to the optimization problem (33), the robust optimization problem is based on the worst-case estimations:

\[ \min_{Q, R} 10 \log_{10} \left\{ \max_{\Delta G_M} \left\| \Delta G_M u_{\text{opt}} \right\|_2 + \max_{\Delta d} \left\| G_M \Delta d \right\|_2 + \max_{\Delta G} \left\| C \Delta G u_{\text{opt}} \right\|_2 \right\}^2 \]

American Institute of Aeronautics and Astronautics
subject to \[ u_{\text{opt}} = -\left[G_0^H Q G_0 + R\right]^{-1} G_0^H Q d_0 + \Delta u_{\text{opt}} \]
\[ Q > 0 \]
\[ R > 0 \]
\[ \min_{i=1}^{N_L} \tilde{\lambda}_i > 0 \]
\[ u_{\text{opt0}} - \max_{i=1}^{N_L} |u_{\text{opt(i)}}| > 0. \]

The optimization variables are still the weighting matrices \( Q \) and \( R \). However, the optimization is now performed for the worst-case approximation. In addition to the nominal plant, the norm of perturbations of \( \Delta G_M \), \( \Delta d_m \), \( \Delta G \) and \( \Delta d \) are necessary.

V. Active Noise Control Experiment

For the active noise control experiment, one load case is considered. The fundamental frequency of the primary noise field is set to 92 Hz. In addition, two higher harmonics are excited at 184 Hz and 276 Hz. The sound pressure level at the artificial head is adjusted to 75 dB(A) for each frequency. This level is chosen to guarantee the linear behavior of the equipment.

The transfer functions and the primary noise fields are measured for the nominal condition. To simulate the perturbation caused by different persons and objects inside the cabin, the transfer functions and primary noise fields are perturbed with three different sound hard diffracting objects suspended on the ceiling of the loadmaster area, see Figure 8. The measured median norm of the uncertainty of the primary noise field and the transfer function is shown in Table 1 for the considered load case. A description of the measurement procedure can be found in Ref. 14.

Subsequently, the controller weighting matrices \( Q \) and \( R \) are optimized according to (33) and (39) using Matlab® with the Genetic Algorithm and Direct Search Toolbox. The nonlinear constraints are included in the nonlinear cost function with the barrier function method. Finally, the controller parameters are copied to the controller. After the controller calibration, the controller is ready to attenuate the primary noise field.

<table>
<thead>
<tr>
<th>Table 1. Referenced median uncertainty norm of the primary noise field and the transfer functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( |\Delta G|_F / |G_0|_F )</td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>0.036</td>
</tr>
<tr>
<td>( |\Delta d|_2 / |d_0|_2 )</td>
</tr>
<tr>
<td>( |\Delta G_{M1}|<em>F / |G</em>{M0}|_F )</td>
</tr>
<tr>
<td>( |\Delta d_{m1}|<em>2 / |d</em>{m0}|_2 )</td>
</tr>
</tbody>
</table>

Figure 8. Diffracting object suspended on the ceiling of the loadmaster area.

Figure 9. Microphones in the monitor volume around the artificial head to evaluate the control performance.
The control profit is measured with the monitor microphones around the artificial head, see Figure 9. The results are shown in Figure 10. Here, a significant energetic mean noise reduction up to 20 dB can be achieved for 92 Hz and 184 Hz. Hereby, the noise reduction of the nominal controller design is higher than the robust controller design method. Only a minor noise reduction is achieved with the nominal design method at 276 Hz. The robust controller design method can achieve no noise reduction. However, with uncertainties the robust controller design method will achieve a more reliable noise reduction.

Figure 10. Sound field mapping around the artificial head for the primary noise field as well as the control performance achieved with nominal controller design and the robust controller design.

VI. Conclusion

In the paper, an experiment is setup to represent the acoustic situation inside a turbo-prop aircraft. Here, the coupling of the small loadmaster area to a large compartment plays an important role. The energy based analysis shows that the primary noise transmits through the coupling surface very well. Therefore, high sound pressure levels are expected due to primary excitation. On the other hand, secondary noise which is generated by the loudspeakers leaks to the cargo hold. Therefore, the loadmaster area will be highly damped despite the sound hard material in the loadmaster area walls. This indicates that the secondary loudspeakers have to provide high capacities. The transfer functions between the secondary sources and control microphones are smooth with low resonance peaks. This is
advantageous for the controll stability. Secondly, the substructure based analysis shows that only a small amount of independent mode shapes influence the vibration behavior inside the loadmaster area. These coupling analysis indicates that an acoustical mock-up of the loadmaster area placed inside a large laboratory room can simulate the most important acoustic effects inside the aircraft.

A prototype active noise control system is installed in the loadmaster area. Here, a steepest descent algorithm is used to control the loudspeaker actuation. Moreover, two new design methods of the optimal parameterization of the controller weighting matrices $Q$ and $R$ are presented. The first design method takes only the nominal case into account. The second design method also considers the uncertainty of the primary noise field and the transfer functions in the design process. First experimental results show the capabilities of these new methods.

In future, the higher robustness of the robust weighting approach will be shown. Moreover, a non-diagonal structure of the weighting matrix $Q$ and $R$ will be analyzed.

References