Two and Three-Dimensional Simulation of Sound Generated by Flow Around a Circular Cylinder

Reinaldo M. Orselli¹, Julio R. Meneghini² and Fabio Saltara³

University of Sao Paulo, POLI, Department of Mechanical Engineering, NDF, Sao Paulo, SP, 05508-970, Brazil

The objective of this paper is to calculate the far-field sound generated from low mach number flow around a two-dimensional and three-dimensional circular cylinder in the subcritical regime using the Lighthill acoustic analogy. For the two-dimensional case, the time-dependent incompressible flow is predicted using unsteady Reynolds-averaged Navier-Stokes models. For the three-dimensional case, the flow was obtained by solving the filtered Navier-Stokes equations of the Large Eddy Simulation model. As a benchmark, a flow-field with a Reynolds number of 90,000 is employed. The obtained numerical results such as Strouhal number, fluctuating lift and mean drag are compared with experiments. The computed unsteady pressure fluctuations on the cylinder wall are used as a sound source for the acoustic solver. Comparison between the two-dimensional numerical results and the experiment shows that computed acoustic field overpredict the noise amplitude; however, good agreement is obtained if an appropriate correlation length is taken into account. The sound measurements obtained by Revell et al. were carried out with a much longer span cylinder length when compared to the span used for the three-dimensional LES simulations. Therefore, the far-field sound is estimated by two correction methods proposed, respectively, by Kato et al. and Seo & Moon. The aerodynamic and acoustic results obtained by the three-dimensional approach agree favorably well with the corresponding experimental data.

Nomenclature

\[ D = \text{cylinder diameter [m]} \]
\[ U_0 = \text{freestream velocity [m/s]} \]
\[ Re = \text{Reynolds number based on cylinder diameter} \]
\[ M = \text{Mach number based on the freestream velocity} \]
\[ St = \text{Strouhal number} \]
\[ Cp = \text{pressure coefficient} \]
\[ Cl = \text{lift force coefficient} \]
\[ Cl' = \text{r.m.s. of the fluctuating lift force coefficient} \]
\[ Cd = \text{drag force coefficient} \]
\[ Cd' = \text{r.m.s. of the fluctuating drag force coefficient} \]
\[ \theta = \text{azimuthal position on the cylinder wall with reference to its the stagnation point} \]
\[ \theta_s = \text{cylinder flow point of separation with reference to its the stagnation point} \]
\[ SPL = \text{sound pressure level [db]} \]
\[ OASPL = \text{overall sound pressure level [db]} \]
\[ L = \text{longer cylinder span length (used in the measurements of Revell et al.) [m]} \]
\[ L_a = \text{representative cylinder span length based on the two-dimensional approach [m]} \]
\[ L_s = \text{simulated cylinder span length [m]} \]
\[ L_c = \text{coherence cylinder span length according to Kato et al. correction method [m]} \]
\[ L_{c'} = \text{coherence cylinder span length according to Seo and Moon correction method [m]} \]
\[ \Gamma_y = \text{aerodynamic spanwise coherence function} \]
\[ \gamma_y = \text{acoustic spanwise coherence function} \]

¹ Research Student, Department of Mechanical Engineering, reinaldo.orselli@poli.usp.br.
² Associate Professor, Department of Mechanical Engineering, jmeneg@usp.br.
³ Associate Professor, Department of Mechanical Engineering, fsaltara@usp.br.
I. Introduction

Due to stricter noise emission regulations, the reduction of noise radiation has been recently a major concern in the design of speed trains, airplanes and road vehicles. An understanding of the physical mechanisms involved in sound generation is a crucial step for reducing or controlling the sound emission. In this paper, the sound generated by a flow over a circular cylinder is simulated numerically. As a benchmark based on the experimental data of Revell et al., a flow with a Reynolds number $Re = 90,000$ and a freestream Mach number $M = 0.2$ was simulated. The simulations were conducted considering two-dimensional (2D) and three-dimensional (3D) computational domains. The flow is characterized by a Von Karman’s vortex street downstream the cylinder involving flow separation and laminar-turbulent transition which makes its simulation a more challenging task. Most of the noise is generated by the unsteady pressure fluctuations on the cylinder wall. For low Mach number flows, the coupling between sound and flow fluctuations can be usually neglected. Thus, the flow which is the source of noise can be computed separately from the acoustic field. Under this assumption, an acoustic analogy is employed to solve the acoustic field. The far-field sound is computed from integral solutions of the wave equation which uses as source term the flow field obtained from the CFD solution.

Following the works of Cox et al., Perot et al. and Gloerfelt et al. in this paper we investigate numerically the flow field and its induced sound generated by a two-dimensional unsteady flow around a circular cylinder with a Reynolds number (based on the cylinder diameter) of 90,000. The Reynolds number is within the range of the subcritical flow regime that is characterized by a laminar boundary layer separation with turbulence transition occurring downstream in the wake. In order to obtain the incompressible flow field, Unsteady Reynolds Average Navier-Stokes (URANS) equations are solved using the FLUENT CFD code. Providing the time history of the flow field noise sources, in a post-processing step, the far field sound is predicted numerically by the integral solution of acoustic analogy equations based on the Ffowcs Williams and Hawkings (FW-H) method presented originally in Ffowcs Williams & Hawkings. Despite the fact of the flow being inherently 3D for $Re = 90,000$, a two-dimensional numerical simulation can capture the main physical aspects of the flow dominated by an alternate and periodical vortex shedding. Therefore, the present article intends to evaluate the ability of the two-dimensional URANS numerical solutions to predict the unsteady flow over the cylinder and its associated noise generated.

In order to solve the flow over the cylinder more appropriately taking into account its 3D effects, a second three-dimensional simulation was carried out using the filtered incompressible Navier-Stokes equations of the Large Eddy Simulation (LES) model. The wall pressure fluctuations over the cylinder span (provided by LES) were used as source to the integral solutions of the wave equation (FW-H analogy) so as to obtain the far field sound pressure. This approach of solving the 3D flow over the cylinder in a high-Reynolds range ($2 \times 10^4$–$1.5 \times 10^5$) in the subcritical regime using LES and obtaining the far-field sound by integral solutions of the wave equation has been already used successfully in many previous works such as Perot et al., Boudet et al., Seo & Moon, Takashi et al. and Kato et al. Finally, as the experiments of Revell et al. were performed with a much longer cylinder span length, the computed far-field sound pressure levels are then estimated for the corresponding longer span by two correction methods proposed, respectively, by Kato et al. and Seo & Moon.

II. Governing Equations and Numerical Methods

In the present work, the far-field sound is predicted by a more general form of the Lighthill acoustic analogy, developed by Ffowcs Williams & Hawkings, which includes the effect of surfaces in arbitrary motion. The near-field source fluctuations of the incompressible flow are first obtained by solving the fluid governing equations (two-dimensional and three-dimensional approaches) whose results are used, as input data, to feed the acoustic computation. Due to the fact that the acoustic analogy separates the flow field and acoustic computations, acoustic and aerodynamic approaches will be, in general, presented separately.

A. Two-dimensional aerodynamic computation methodology

Computational fluid dynamics (CFD) is used to obtain the unsteady flow field. Applying a time averaging (or Reynolds-averaging) to the Navier-Stokes equations (RANS), an additional term, the Reynolds stress tensor, appears which is modeled by making use of the Boussinesq eddy-viscosity approximation. In this assumption the Reynolds stress tensor is computed as product of an eddy viscosity and the main strain rate tensor, Wilcox. In order to close the system of equations, this eddy turbulent viscosity is related to additional scalar turbulent quantities that are solved by its own transport equations. The turbulence models are usually classified by the number of these added transport scalar equations. It the present paper, the RANS two-equation $k-\omega$ SST (SST - shear stress transport) model of Menter is used. It is a hybrid model that makes use of the standard $k-\epsilon$ model (Launder & Spalding), equations in the fully turbulent region (far from the walls) and transforms into the $k-\omega$ equations model (Wilcox).
close to wall region. In addition, the eddy viscosity definition is modified to account for the transport of the principal turbulent shear stress. This model gives good results for flows characterized by zero pressure gradient and adverse pressure gradient boundary layers (Versteeg & Malalasekera\textsuperscript{15}) which makes it more appropriate for the problem of the flow over the cylinder. In addition, as a reference, two other classical turbulence models are compared to the $k-\omega$ SST model, the $k-\varepsilon$ model and the one-equation Spalart-Allmaras model, S-A, (Spallart & Allmaras\textsuperscript{16}), which is a model widely used for airfoil applications in the aerospace and turbomachinery community. In order to use the Reynolds-Averaging of Navier-Stokes equations (RANS) appropriately in unsteady turbulent flows, the time interval should be shorter than the time variations of the main flow structures of interest (for instance, vortex shedding period of the flow over the cylinder) but much higher than the characteristic time scale of the random fluctuations associate with turbulence. In this case, only the main unsteady flow structures may be well resolved by RANS turbulent models. If ones desire more details of turbulent motions either Direct or Large Eddy Simulation can be used.

In order to predict the boundary layer and separation around the cylinder wall, the approach of resolving the near wall-region (viscosity-affected inner region) is used in all employed turbulence models. For the near wall model treatment, the two-layer model originally proposed by Wolfshtein\textsuperscript{17} is applied. In this model, the computational domain is subdivided into a near-wall viscosity-affected region and a fully-turbulent region. In the viscosity-affected region, the turbulent eddy viscosity is related to the turbulent kinetic energy ($k$) and a near-wall adapted length scale, whose formulation was proposed by Chen & Patel\textsuperscript{18}.

A low-Reynolds-number correction to the turbulent viscosity in the standard $k-\omega$ model was proposed by Wilcox\textsuperscript{9}. This modification was developed to take into account both the viscous near-wall effects and laminar-turbulent transition at boundary layers flows. This correction was extended to the $k-\omega$ SST model which is used, in the present paper, for solving the fluid flow over the cylinder. This model is referred here as $k-\omega$ SST transition.

All the numerical simulations were performed using the FLUENT CFD finite-volume code. The numerical discretization scheme employed for the pressure-velocity coupling was the PISO algorithm (originally developed for unsteady compressible flows, see Versteeg & Malalasekera\textsuperscript{15} for more details). The upwind second-order spatial differencing method was applied for the convective terms. The solution is time-advanced using an implicit and second-order accurate scheme.

The cylinder diameter ($D$) is 0.019 m and the freestream Mach number employed is $M = 0.2$ corresponding to $Re \approx 90,000$ (flow parameters used in the experiment of Revell et al.\textsuperscript{1}). A structured form mesh with 69,699 cells is employed. The computational domain top and bottom boundaries were both located at 10.5$D$ from the cylinder axis. The inlet and outlet exit were placed, respectively, at 8.5$D$ and 20.5$D$ from the cylinder axis whose values were based on the articles by Kim\textsuperscript{19-20}. The cylinder surface is discretized with 240 volume cells. In order to solve the near-wall flow, a spatial mesh resolution of $\Delta y^+ \approx 1 (= u_\tau \Delta y / \nu)$ was employed on the cylinder wall. For adequate temporal resolution, each shedding cycle was divided by approximately 234 time steps. Figure 1 shows the computational mesh used in the two-dimensional simulations.

![Figure 1. Computational structured mesh used for the 2D CFD simulations, 69,699 volumes cells.](image)

**B. Three-dimensional aerodynamic computation methodology**

For the three-dimensional CFD computations, in order to solve the flow over the cylinder, the Large Eddy Simulation (LES) turbulence model was used with the FLUENT code. The computational domain size in the x-y plan (see Fig. 2) was the same that used for the 2D calculations. The choice of the computational domain size in the
spanwise direction ($L_z$) should be chosen carefully as it can affect significantly the results. In addition, the mesh resolution along the span direction is another important aspect which can exert a significant influence on the results. According to Kim\textsuperscript{19}, considering the use of periodicity condition between the two lateral boundaries in the spanwise direction, the length of the computation domain (spanwise direction) should be large enough to take into account most of the turbulence correlation length over a range of frequencies which can considerably affect the computational results. For a cylinder flow with $Re = 90,000$, based on experimental data, Norberg\textsuperscript{21} provided a spanwise correlation length of $A = 3.16D$ (based on the fluctuating lift forces) whose value can be used as a reference spanwise length to be used on the computation domain. Moreover, for $Re = 1.4 \times 10^5$, using a spanwise length of $2.0D$ the LES results of Breuer\textsuperscript{22} and Kim\textsuperscript{19,20} agreed fairly well with experimental data. Considering the above-mentioned information and taking into account computational power limitations, a computational domain size in the spanwise direction of $2.5D$ is used in all LES present computations whose size domain may at least capture part of the flow three-dimensionalities.

With regard to grid resolution requirements near the wall, the mesh was clustered to resolve the viscous sublayer of the near-wall region which corresponds to $y^+$ below 1.0 (no-slip condition). In addition, LES computations require finer and adequate mesh resolution for obtaining reliable results, high aspect ratio grid cells should be avoided (Travin \textit{et al.}\textsuperscript{23}) and the grid should be fine enough to well resolve smaller turbulent eddies (Pope\textsuperscript{24} recommends to resolve 80% of the energy containing range of the flow eddies). In practical terms, due to computation power limitation, it is not possible to follow the recommendations above-mentioned over all computation domain. In the case here, the grid was refined especially in the region around the cylinder wall (at least up to twice the cylinder diameter) and over the cylinder wake where the highest flow fluctuations are observed. As a target, the mesh was built in order to have a nearly cubic grid cell along the region $0.75D$ away from the cylinder center axis. Consequently, 112,000 cells were used in the main plan ($x$-$y$) and in the spanwise direction 48 cells were employed, which gives a total of 5.4 millions cells (hexahedral type), see Fig. 2. The cylinder surface were discretized with 310 volumes cells.

A condition of a zero-shear slip wall was applied at the top and bottom boundaries. A constant freestream velocity ($U_\infty$) with zero turbulence level was employed at the upstream inlet boundary. At the downstream exit boundary, a constant freestream pressure was imposed. A spanwise periodicity condition was applied for the two lateral boundaries.

Particularly, with regard to the LES model, the subgrid scale was modeled by the Dynamic Smagorinsky Model (DSM) based on the proposals of Germano \textit{et al.}\textsuperscript{25} and Lilly\textsuperscript{26}. Details of the LES model based on a finite-volume solver implemented in FLUENT are found in Kim\textsuperscript{20,27}. In order to avoid numerical diffusion in LES, the convective fluxes can be discretized by a non-dissipative central-differencing scheme (CD). In the present LES simulation, the bounded central differencing (BCD) scheme available in the FLUENT package was applied. In essence, the BCD scheme avoids possible unphysical numerical oscillations of the CD scheme, while preserving its non-dissipative properties, more details in Kim\textsuperscript{20}. The discretized transport equations are advanced in time by using the NITA (Non-Iterative Time-Advancement) algorithm where the costly outer iterations are only performed once at each-time step while preserving the overall time accuracy. In addition, within NITA scheme, the momentum equations are decoupled from the continuity equation using an approximate factorization of the Navier-Stokes equations.
(fractional-step method). This non-iterative approach based on the fractional-step method provides a highly cost-effective method for solving unsteady flows such as LES. For temporal-discretization, an implicit three-level second order accurate scheme is applied. The non-iterative fractional step method combined with the temporal-discretization scheme implemented in FLUENT are fully described in Kim28.

The time-step size ($\Delta t$) was determined in function of the smallest resolved eddies considered for the flow calculations, it was used a $\Delta t$ corresponding to 1/420 times the typical vortex shedding cycle ($\Delta t = 0.0125$ in dimensionless unit, $D/U_\infty$). The estimation proposed in Kim20 where the smallest resolved eddy size would be roughly $\ell = 0.05D$ with a characteristic velocity of about $u^* = 0.2U_\infty$, a dimensionless time step of $\Delta t = 0.0125$ would resolve one turnover of the smallest resolvable eddies with 20 time steps.

The LES computations were performed on a 16-CPU cluster using Intel quad-core Xeon processors. The 16-CPU took approximately 60 seconds to calculate the flow for each time step which means roughly 35 days to complete 50,000 time steps.

C. Acoustic Computation Methodology

In the acoustic analogy approach, the obtained near-field flow is used as a sound source input into the wave equations to predict the mid-to-far-field noise. It considers that the fluid flow dynamics is completely uncoupled from the acoustic field. The method used here is based on the Ffowcs Williams and Hawking equations and its integral solution. This model is applicable only to predict the propagation of sound toward the free space (surrounded by a uniform fluid flow at rest). It does not take into account wave reflection or scattering due any additional obstacle (solid surface) between the sound flow-field source and the observer.

The Ffowcs Williams and Hawking equations can be obtained by just manipulating the continuity and Navier-Stokes equations (more details in Ffwocs Williams & Hawking5 and Brentner & Farassat29). Basically, it is an inhomogeneous wave equation:

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial}{\partial t} \left[ \left( \rho \hat{u}_n + \rho (u_n - u_n) \right) \delta(f) \right] - \frac{\partial}{\partial x_i} \left[ \left( P_{ij} \hat{u}_j + \rho u_i (u_n - u_n) \right) \delta(f) \right] + \frac{\partial^2}{\partial x_i \partial x_j} \left[ T_{ij} H(f) \right]$$

where $p = p - p_\infty$, $u_i$ and $\hat{u}_i$ are, respectively, the fluid and surface velocity component in the $x_i$ direction, $u_n$ and $\hat{u}_n$ are, respectively, the fluid and surface velocity component normal to the surface ($f = 0$). $\delta(f)$ indicates the Dirac delta function and $H(f)$ correspond to the Heaviside function. A mathematical surface ($S$) is defined by the function $f$, $f < 0$ represents a region inside $S$, $f = 0$ denotes the surface $S$ and $f > 0$ correspond to an unbounded space outside $S$. $n_i$ is a unit normal vector pointing towards outside $S$, $c_\infty$ is the freestream speed of sound and $T_{ij}$ is the Lighthill stress tensor, which is given by,

$$T_{ij} = \rho u_i u_j + P_{ij} - c_n^2 (\rho - \rho_n) \delta_{ij}$$

where,

$$P_{ij} = p \delta_{ij} - \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - 2 \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]$$

In Eq. (1) we have a wave equation with three inhomogeneous acoustic source terms on its right hand side known, respectively, as monopole, dipole and quadrupole sources. The monopole source represents the noise generated due to mass fluctuation of the fluid by moving surfaces. The dipole source corresponds to noise generated by fluctuating forces on the body surface. The quadrupole source accounts for the noise generation due to off-body fluctuating stresses of the fluid.

The solution of Eq. (1) is obtained by applying the convolution product using the free-space Green function $G(g) = \delta(g) / 4\pi r$, where $g = r - r' - c \tau$ with $t$ and $\tau$ being, respectively, the observer and source times. The complete solution is expressed by surface and volume integrals. In this approach, the surface integrals and dipole acoustic sources. The volume integrals represent the quadrupole sources outside the source surface. For flows with low Mach number the volume integrals contribution become small. In addition, if the source surface is placed so that it surrounds all sound sources and its non linear effects, then, it can be shown that all...
The quadrupole noise generated will be represented by the surface integrals. In this case the volume integral should be dropped. The volume integral calculations are very time consuming, thus, in all results, this term is neglected.

\[
\frac{\partial p}{\partial x} (\vec{x}, t) = p_T (\vec{x}, t) + p_L (\vec{x}, t)
\]

\[
4\pi p_T (\vec{x}, t) = \int_{f=0}^{\infty} \left[ \frac{\rho_\infty (U_n + U_\infty)}{r (1-M_r)^2} \right]_{\text{ret}} dS + \int_{f=0}^{\infty} \left[ \frac{\rho_\infty U_n \left( r M_r + c_\infty (M_r - M_e^2) \right)}{r^2 (1-M_r)^3} \right]_{\text{ret}} dS
\]

\[
4\pi p_L (\vec{x}, t) = \int_{f=0}^{\infty} \left[ \frac{L_r}{r (1-M_r)^2} \right]_{\text{ret}} dS + \int_{f=0}^{\infty} \left[ \frac{L_r - L_M}{r^2 (1-M_r)^3} \right]_{\text{ret}} dS + \int_{f=0}^{\infty} \left[ \frac{L_r (r M_r + c_\infty (M_r - M_e^2))}{r^2 (1-M_r)^3} \right]_{\text{ret}} dS
\]

In Eq. (3), new variables \( U_i = [u_i + \rho f \rho_\infty (u_i - u_\infty)] \) and \( L_i = \rho f \rho_\infty (u_i - u_\infty) \) are introduced, where \( U_n = \vec{U} \cdot \hat{n} = U_n \hat{n} \) and \( L_n = \vec{L} \cdot \hat{n} = L_n \hat{n} \), \( \hat{n} \) and \( \hat{r} \) correspond to unit vectors, respectively, in the surface normal direction and in the radiation direction. The square brackets with the subscript \( \text{ret} \) denote that the integrands are computed at the retarded time when the sound was emitted, \( \tau = t - r/c_\infty \), where \( t \) is the time at the observer and \( r \) is the distance between the sound source and the observer. The dot over the variable represents source-time differentiation of the variable (i.e., \( U_n = U_n \hat{n} / \hat{c} \tau \)). Also, \( \hat{M} \) is a local Mach number vector with components \( M_i \), where \( M_r = \hat{M} \cdot \hat{r} = M_r \hat{r} \) and \( L_M = L_r \hat{r} \).

When the source or integration surface is placed on the body (impermeable wall), the terms, in Eq. (3), \( p_T (\vec{x}, t) \) and \( p_L (\vec{x}, t) \) are known as thickness and loading noise, respectively, related to its physical meanings. In this formulation, the source surface \( (f = 0) \) does not have to be placed on body surfaces or walls. The integration surface can be located in the interior flow (off body surface). In this case the surface is permeable and the surface integrals of Eq. (3) account for the contributions of the quadrupole noise sources inside the region enclosed by the source surface. It should be noted that within the permeable surface, the grid resolution needs to be fine enough to resolve all significant unsteady flow structures including sound propagating waves. This specific approach is known as porous FW-H formulation. In the present work, all acoustic results were obtained utilizing the software FLUENT.

### III. Aerodynamic Results

In order to obtain the acoustic far-field, the near-field unsteady flow results is used as an input data to the wave equations, Eq. (1). Thus, the noise prediction depends directly on the accuracy of the CFD results. In this paper, the two-dimensional unsteady versions of the turbulence models \( k - \omega \) SST transition, \( k - \omega \) SST, S-A and \( k - e \) are used for obtaining the time-history of the near-field flow over a circular cylinder at \( Re = 90,000 \). In all 2D simulations, it was investigated the mesh refinement effects on the numerical results, no more refinement was considered to be necessary for a mesh with 69,669 volumes, Fig. 1. In addition, a three-dimensional LES simulation was performed whose results are compared with the two-dimensional computations.

The accuracy of the numerical simulations is generally evaluated by comparing it with available experimental results. For bluff bodies, mean flow quantities such as mean drag coefficient \( (C_d) \) and angle of flow separation \( (\theta_f) \) and, also, fluctuating quantities such as shedding frequency \( (St = f D/U_\infty) \) and r.m.s. of fluctuating drag and lift coefficients (respectively, \( C_d \) and \( C_l \)) are commonly used as benchmark parameters for evaluating the quality of the CFD results in comparison with its corresponding available experimental data. Table 1 shows the obtained results of the two-dimensional (URANS models) and the three-dimensional (LES) numerical simulations which are compared to the corresponding experimental data.
In Table 1, it can be noted that each turbulence model produced quite different results for all flow quantities. Concerning the two-dimensional URANS models, the highest values of r.m.s of $l_C$, mean and r.m.s. of $d_C$ were obtained by the $k - \omega$ SST transition model. The results show that the values of $C_l$ and $C_d$ are very sensitive to the choice of the turbulent model. In comparison with the experimental data, all two-dimensional simulations predicted a slightly higher shedding frequency whereas the 3D LES simulation agrees well. The overprediction of the shedding frequency from two-dimensional computation is an expected CFD result. As argued by Casalino, in 2D simulations the mean Reynolds stresses are higher (in comparison to 3D flows) corresponding to shorter mean recirculating regions, as a consequence, the recirculating regions are slightly closer resulting in a higher Strouhal frequency. With regard to the two-dimensional simulations, it can be observed a direct relation of the lift and drag fluctuations and mean drag with the point of flow separation. The farther the point of separation occurs, the lower are the fluctuating forces and the mean drag. Thus, the ability of the turbulence model to predict separation determines much of the flow behavior around the cylinder. Comparing the $k - \omega$ SST transition and $k - \omega$ SST, which differs only from their near-wall approach, it can be noted the superior performance of the low-Reynolds near-wall approach of Wilcox dealing with separating flows. The fact that this near wall approach takes into account local laminar-turbulent transition may have contributed to emulate the flow separation physics. Therefore, the most accurate 2D turbulence model is the $k - \omega$ SST transition, despite its higher $C_l$ values which is mostly related to the fact that the 2D simulation considers the vortex shedding fully correlated along the cylinder span.

Regarding the 3D simulation, results in Table 1 show the capability of LES model to well predict, within the experimental data range, most of the main global parameters of the flow over the cylinder in the subcritical regime, except for $C_d$. Despite its high computational cost, LES is designed to resolve directly the time-dependent large eddies and model the smaller eddies, which tend to have a universal behavior (nearly isotropic). Thus, this approach allows solving the flow more accurately and provides more detailed flow information particularly related to the statistics of the resolved fluctuations, which makes LES an adequate tool for aeroacoustics applications in comparison with URANS based approaches.

The time history of the aerodynamic forces (lift and drag) acting on the cylinder obtained by the URANS model $k - \omega$ SST transition model is presented in Fig. 3a. It can be observed that the 2D URANS model predicts an almost perfectly periodic flow which indicates the presence of a fully spanwise correlated main vortex shedding. The Strouhal number corresponding to the main shedding frequency can be obtained by spectral analysis of the lift fluctuations shown by the maximum peak in Fig. 3b. Moreover, in Fig. 3b, it is noted peaks at odd harmonics frequencies ($f_0$, $3f_0$, $\ldots$) for the unsteady lift spectrum and peaks at even harmonics ($2f_0$, $4f_0$, $\ldots$) for the unsteady drag spectrum. Thus, the aerodynamic forces acting in the streamwise direction fluctuates at twice the frequency of those in the vertical direction, but the amplitude of the vertical fluctuating forces are much more significant than the forces in the streamwise direction.

In Fig 4a, the time histories obtained by LES of the lift and drag forces are shown. In comparison with the two-dimensional computations, an amplitude modulation of the lift forces is observed which is mainly due to the occurrence of irregular vortex break-up along the cylinder span leading to a three-dimensional vortex shedding. In addition, it is noted that the drag forces fluctuate randomly with a much lower fluctuating level in comparison with the lift forces. The power spectral density (PSD) of the fluctuating lift forces is plotted in Fig. 4b. Levels of PSD are observed all over the spectrum with a distinguished peak at the vortex shedding frequency corresponding to $St = 0.19$. 

---

**Table 1. Experimental and obtained numerical results of $St$, $C_l$, $C_d$, $\theta$, and $\theta_l$.**

<table>
<thead>
<tr>
<th></th>
<th>$St$</th>
<th>$C_l$</th>
<th>$C_d$</th>
<th>$\theta_l$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>0.180-0.191</td>
<td>0.45-0.60</td>
<td>1.0-1.4</td>
<td>0.18</td>
<td>80º</td>
</tr>
<tr>
<td>data</td>
<td>Norberg²¹</td>
<td>Norberg²¹</td>
<td>Cantwell &amp; Coles³⁰</td>
<td>West²³</td>
<td>Achenbach³²</td>
</tr>
<tr>
<td>$k - \omega$ SST transition</td>
<td>0.235</td>
<td>0.823</td>
<td>1.09</td>
<td>0.062</td>
<td>88º</td>
</tr>
<tr>
<td>$k - \omega$ SST</td>
<td>0.247</td>
<td>0.762</td>
<td>0.944</td>
<td>0.061</td>
<td>99º</td>
</tr>
<tr>
<td>S-A</td>
<td>0.242</td>
<td>0.165</td>
<td>0.625</td>
<td>0.0031</td>
<td>96º</td>
</tr>
<tr>
<td>$k - \omega$</td>
<td>0.282</td>
<td>0.090</td>
<td>0.479</td>
<td>0.0013</td>
<td>109º</td>
</tr>
<tr>
<td>3D LES</td>
<td>0.191</td>
<td>0.485</td>
<td>1.08</td>
<td>0.077</td>
<td>81º</td>
</tr>
</tbody>
</table>
In Fig. 6a, the mean pressure coefficient ($C_p$) distribution along the cylinder surface obtained by each simulated turbulence model is compared with the experimental data of Cantwell & Coles\textsuperscript{30}. None of the 2D URANS simulations is able to predict the measured $C_p$ distribution, except on the back of the cylinder ($\theta > 120^\circ$) whose values obtained by the $k - \omega$ SST transition $Z$ and $k - \omega$ SST model are equivalent to the experiment. In addition, the plot shows that the 3D LES results are mostly comparable with the experimental $C_p$ distribution all around the cylinder surface. The lowest peak predicted by LES is slightly underpredicted and the pressure on the back surface ($\theta > 150^\circ$) is moderately higher than the measurements.

Fig. 6b shows the mean dimensionless skin-friction coefficient given by each simulated turbulent model which is compared with the experimental data of Achenbach\textsuperscript{32} obtained at a slightly higher Reynolds number of 100k. The flow separation point ($C_f = 0$) is only well predicted by the LES model. The 2D URANS model which best fit the measurements is the $k - \omega$ SST transition model. In general, the LES results are equivalent to the experimental data, however around the highest peak and soon after the flow separation the skin friction is slightly overpredicted.

A qualitative view of the numerical results obtained by the two-dimensional $k - \omega$ SST transition model is shown in Fig. 7 where the vorticity magnitude contours at an instant of time are presented. It is observed that the flow is mostly characterized by a great alternating vortex shedding.

In order to show the three-dimensional flow structures in the cylinder wake provided by LES, the iso-surfaces of the second invariant of the velocity deformation tensor (Q-criterion), colored by the resolved turbulent kinetic energy, is illustrated in Fig. 8. We can still visualize coherent alternating vortex shedding in the cylinder wake. The colored iso-surfaces indicate that just downstream the boundary layer separation the flow go through a transition to turbulence at the shear layer in the cylinder wake.
Figure 6. (a) Azimuthal distribution of mean static pressure coefficient on the cylinder; (b) Azimuthal distribution of scaled mean skin-friction coefficient on the cylinder surface.

Figure 7. Flow field view of the vorticity magnitude contours obtained by the two-dimensional $k - \omega$ SST transition model, unit [m$^{-1}$].

Figure 8. Flow structure view behind the 3D circular cylinder (LES model). Iso-surfaces of the second invariant of the velocity deformation tensor, Q-criterion of $-2.6 \times 10^7$, colored by the resolved turbulent kinetic energy [m$^2$/s$^2$].
IV. Acoustic Results

For sound calculation, the experiment performed by Revell et al.\textsuperscript{1} which consisted of a free jet flow over a circular cylinder placed inside an anechoic wind tunnel was used as a comparison case (benchmark). In the experiment, a cylinder diameter and span length of, respectively, \(D = 0.019\) m and \(L = 25.3D\) was employed. The microphone was located at \(128D\) away from the cylinder axis and positioned at an angle of \(\theta = 90^\circ\) from the cylinder stagnation point. The tests were conducted with a flow Mach number of \(M = 0.2\) and Reynolds number of \(Re = 89,000\). In this work, the sound is computed from the unsteady CFD results considering two-dimensional and three-dimensional computation domains. Therefore, the results for each approach, two-dimensional and three-dimensional domains, are presented below in two different subsections.

A. Two-dimension approach

In this section, it is presented the predicted far-field sound computed by integral solution of the wave equation, Eq. (3), using the two-dimensional CFD unsteady results as input data. In all acoustic computations presented in this subsection, the flow field is obtained by the \(k-\omega\) SST transition model. The experimental data of Revell et al.\textsuperscript{1} was chosen to be compared to the acoustic numerical results.

Regarding the far-field noise computed at the observer (128D way from the cylinder), 8192 acoustic pressures were stored covering approximately 70 vortex shedding cycles, which permitted a greater spectrum resolution.

The acoustic results are presented in terms of sound pressure level in the decibel scale (dB). The sound pressure level (SPL) is defined as,

\[
SPL = 20\log\left(\frac{P_e}{P_{ref}}\right)
\]

where \(P_e\) is the effective sound pressure which corresponds to its root mean square and \(P_{ref}\) denotes a reference pressure, it is used a value of 20 \(\mu\)Pa in air. For the sound spectrum, SPL values are plotted against its corresponding frequency. In this case, the SPL is computed considering the effective pressure (\(P_e\)) as the amplitude spectrum of the fluctuating acoustic pressure data. Using the acoustic spectrum results, an overall pressure sound level (OASPL) can be obtained by adding all noise amplitudes of the spectrum, the OASPL can be obtained applying the following expression:

\[
OASPL = 20\log\sqrt{\sum_{i=1}^{n} (10^{SPL_i/20})^2}
\]

Due to the low Mach number (\(M = 0.2\)), the contribution of quadrupole sources is not very significant, thus most of the sound is generated on wall surfaces (dipole and monopole sources). Therefore, the acoustic spectrum was here computed considering all noise sources being generated on the cylinder wall surface.

In order to use the two-dimensional CFD results as an input data for the acoustic computations, flow data over the span direction is needed as the integration surfaces of the wave equation are performed over a three-dimensional cylinder. In this case, the flow is assumed perfectly correlated over all the cylinder span length which means that the surface integrals are evaluated over the span length assuming source noise data identical to the obtained from the basic 2D simulations. As a consequence the noise prediction is overpredicted as for \(Re = 90,000\) the vortex shedding is inherently three-dimensional (not fully correlated over the span). If it is assumed that weakly correlated flow (over the cylinder span) does not contribute significantly to the overall sound, it is possible to use an acoustic representative span length, \(L_a\), over which the flow is assumed perfectly correlated contributing itself to all sound generated and noise sources beyond \(L_a\) are neglected. Based on the strategy employed by Cox and Brentner\textsuperscript{2}, the acoustic representative span length (\(L_a\)) can be varied in order to evaluate its effect on the overall sound noise level (OASPL). The value of the fluid spanwise correlation length (\(\Lambda\)), defined in Norberg\textsuperscript{21}, can be used as a first estimation for evaluating \(L_a\). For \(Re = 90,000\), Norberg\textsuperscript{21} provides an experimental value of \(\Lambda = 3.16D\). Here, the OASPL was computed for five different acoustic representative span lengths, \(L_a\) : 2.5D, 3.16D, 5.0D, 10.0D and 25.3D. According to the results showed in Table 2, the length \(L_a\) which best fitted the experimental results of OASPL was approximately 5.0D. Providing 2D flow data from CFD simulations, the value of \(L_a = 5.0D\) can be
used as a representative length which at least matches the acoustic level generated by the fully three-dimensional flow over the cylinder with \( L = 25.3D \). Fig. 9 presents the acoustic spectrum obtained for each \( L_a \) (2.5\( D \), 3.16\( D \), 5.0\( D \), 10.0\( D \) and 25.3\( D \)) which is compared with the corresponding experimental result of Revell et al.\(^1\). It is observed that the two-dimensional approach can only predict discrete values of \( SPL \) associated with the fundamental frequency (Strouhal number) and its harmonics.

**Table 2.** Effect of acoustic representative span length, \( L_a \), on the \( OASPL \) results. Revell et al.\(^1\) measured a \( OASPL \) of 100 dB.

<table>
<thead>
<tr>
<th>( L_a )</th>
<th>Measurements Revell et al.(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5( D )</td>
<td>94.9</td>
</tr>
<tr>
<td>3.16( D )</td>
<td>96.9</td>
</tr>
<tr>
<td>5.0( D )</td>
<td>100.9</td>
</tr>
<tr>
<td>10.0( D )</td>
<td>106.9</td>
</tr>
<tr>
<td>25.3( D )</td>
<td>114.9</td>
</tr>
<tr>
<td></td>
<td>100.0</td>
</tr>
</tbody>
</table>

**Figure 9.** Comparison of sound pressure level spectra for five different acoustic representative span length (\( L_a \)) with the experimental results of Revell et al.\(^1\). Microphone located 128\( D \) away from the cylinder, positioned 90° from the stagnation point.

If not indicated, all calculations were performed considering an on-body integration surface which means that the quadrupole noise sources were neglected. Despite the fact that the flow was simulated as incompressible, the effects of an off-body integration surface on the far-field sound pressure was analyzed. An off-body integration surface size of 2.0\( D \) diameter (see Fig. 10) was considered for two different mesh resolutions, respectively, with 69,699 and 152,285 volume cells. Table 3 compares the obtained results of \( OASPL \) using on-body and off-body integration surfaces. Little difference between the acoustic results is observed which indicates that the use of off-body integration surface of size around 2.0\( D \) does not affect significantly the overall noise level (as long as an adequate mesh resolution is employed). Moreover, comparison of the results obtained with the two different meshes on-body and off-body surfaces (see Table 3) demonstrates that the grid resolution exerts some influence on the noise level which may be due to numerical diffusion.
Regarding sound computations from 3D flow simulations, the experimental acoustic results were obtained for a cylinder of span length $25.3D$ whereas the noise computed, using Eq. (3), from the CFD results were performed considering a cylinder of length $2.5D$. In this case, in order to take into account the additional sound level generated due the longer span of the cylinder, two acoustic correction methods are used, the one proposed by Kato et al. and more recently the method developed by Seo & Moon. In general, the correction method approach consists of estimating a $SPL$ correction ($\Delta SPL_{corr}(\omega)$) to be added so as to account for the longer span. Sound correction methods due to longer span bodies have already been used successfully in many others studies, see Refs. 34-37.

In this work, with regard to noise computations from 3D CFD results, 22,000 acoustic pressures values were stored (every time step) corresponding approximately to 50 vortex shedding cycles.

Depending on the frequency ($\omega$) we have different degrees of coherence of the surface pressure fluctuations along the cylinder span. This way, Kato et al. proposes the following formulation:

$$\begin{align*}
L_c(\omega) \leq L_z, & \quad \Delta SPL_{corr}(\omega) = 10\log[L/L_z] \\
L_z < L_c(\omega) < L, & \quad \Delta SPL_{corr}(\omega) = 20\log[L_c(\omega)/L_z] + 10\log[L/L_c(\omega)] \\
L \leq L_c(\omega) & \quad \Delta SPL_{corr}(\omega) = 20\log[L/L_z]
\end{align*} \tag{6}$$

where $L_z$ is the cylinder span length used in the CFD simulations, $L$ is the longer span length whose sound correction is to be estimated and $\Delta SPL_{corr}(\omega)$ is the estimated sound pressure level correction to be added due the longer span. According to Kato et al., the degree of coherence of the fluctuating surface pressure along the cylinder span can be represented by an equivalent coherence length, $L_c(\omega)$, which is the only unknown value of the correction method. The equivalent coherence length, $L_c(\omega)$, can be obtained by calculating the coherence function $\Gamma_{xy}(\omega)$ of the fluctuating wall pressure which is given by:

![Figure 10. Mesh representation showing the off body integration surface (diameter 2.0D). Meshgrid with 69,699 cell volumes.](image)

Table 3. $OASPL$ results with on body and off body integration surfaces for grids with 69,699 and 152,285 volumes cells. Acoustic representative span length of 5.0D.

<table>
<thead>
<tr>
<th>Grid cell volumes</th>
<th>Integration surface</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>69,699</td>
<td>on cylinder</td>
<td>off cylinder (2.0D)</td>
</tr>
<tr>
<td>152,285</td>
<td>101.12 dB</td>
<td>101.31 dB</td>
</tr>
</tbody>
</table>

The table shows the $OASPL$ results for different grid cell volumes with on body and off body integration surfaces. The difference in $OASPL$ values is also provided.
\[ \Gamma_{ij}(\omega) = \frac{\text{Re}\{\hat{P}_i \hat{P}_j^*\}}{\sqrt{\vert\hat{P}_i\vert^2 \vert\hat{P}_j\vert^2}} \]  

(7)

where \( \hat{P}_i \) and \( \hat{P}_j \) are pressure fluctuations (frequency domain) at different span positions on the cylinder surface. In fact, the coherence function is a way of calculating an average phase lag between \( \hat{P}_i \) and \( \hat{P}_j \) located at different span positions on the cylinder surface. This function should take a value of unity if the two fluctuating pressures are perfectly correlated. Otherwise, this function should take a value of zero if the two pressures are fluctuating in a completely independent way (totally uncorrelated). As the spanwise distance between the points \( i \) and \( j \) (\( \Delta z_{ij} \)) increases the values of \( \Gamma_{ij}(\omega) \) tends to decrease. Thus, as a criteria, proposed by Kato et al.\(^{10}\), the corresponding spanwise distance whose value of the coherence function drops to one half determines the value of \( L_c(\omega) \). As the main wall pressure fluctuations occurs around \( \theta = 90^\circ \) and the observer is located \( 128D \) away above the cylinder axis, in the present study, the coherence function is calculated using the fluctuating pressure values at the top surface wall of the cylinder (\( \theta = 90^\circ \)), whose values are provided by the 3D CFD simulations. The time-dependent pressure surface was monitored in 48 points equally distributed along the cylinder span length during the simulation. The pressure phase lag (or \( \Delta z_{ij} \)) between each two different points were calculated limited to half the simulated cylinder span length due to the periodic boundary condition applied on the two lateral boundaries of computation domain. The obtained values of the coherence function were all ensemble averaged according to the spanwise distance between each point (\( \Delta z_{ij} \)). The obtained coherence function of three Strouhal numbers (\( St = 0.19, 0.38 \) and 0.76) are showed in Fig. 11 (symbols) whose values are only valid at most up to half the simulated cylinder span (\( L_c/2 \)). At the Karman vortex shedding frequency (\( St = 0.19 \)) the coherence function decays very slowly being close to unity whereas at the other two frequencies the coherence rapidly decreases as the \( \Delta z_{ij}/D \) increases. If the calculated coherent function does not drop to one half within the valid span length, Kato et al.\(^{10}\) proposes the use of extrapolation based on the obtained coherent function values to determine the coherence length, \( L_c(\omega) \). Therefore, in order to obtain the coherent length at vortex shedding frequency, \( L_c(\omega) \), the obtained coherence function are fitted and extrapolated by a Gaussian function, \( \exp(-\Delta z^2/L_c(\omega)^2) \) (solid line in Fig.11), as proposed by Seo & Moon\(^7\). At \( St = 0.38 \) and 0.76 the calculated coherence function is also fitted by a Gaussian function which is used to obtain its corresponding coherent length, \( L_c(\omega) \). In Fig. 11, the Gaussian function seems to fit well the coherence values at the Karman vortex shedding frequency. However, at \( St = 0.38 \) the Gaussian function does not represent well the calculated values, even though the corresponding coherence function results obtained by Seo & Moon were well fitted by the Gaussian function (in the reference a 3D flow over a circular cylinder with \( Re = 46,000 \) was simulated using LES, \( L_c = 3.0D \)). Casalino & Jacob\(^{33}\) considers the possibility of using either a Gaussian or a Laplacian function for representing the coherence function decay with the increase of the spanwise distance as both are consistent with their statistical assumptions. Moreover, Perot et al.\(^3\) employed Laplacian functions, \( \exp(-\Delta z/L_c(\omega)) \), to fit their calculated coherence functions (in this work\(^3\) the flow around a 3D cylinder was obtained by LES, \( Re = 1.4 \times 10^5 \), \( M = 0.16 \) and \( L_c = 1.5D \)). Thus, at \( St = 0.38 \) and 0.76 the coherence values are also fitted by a Laplacian function whose results are plotted in Fig. 12. Particularly for \( St = 0.38 \) neither the Gaussian nor the Laplacian function seemed to well represent the coherence function decay with the increase of spanwise distance. On the other hand, at \( St = 0.76 \) both Gaussian and Laplacian function fitted relatively well with the calculated coherence functions. Table 4 compares the obtained correlation lengths, according to Kato et al.\(^{10}\) criteria, using either Gaussian and Laplacian fits. At \( St = 0.38 \) and 0.76, no significant difference is observed comparing the correlation length obtained with a Gaussian or a Laplacian fit. At the Karman vortex shedding frequency, the coherence function does not fit at all with a Laplacian function, thus no result is shown in Table 4.
Figure 11. Spanwise coherence function of the top wall pressure obtained by the 3D LES computations ($\theta = 90^\circ$). Data are fit with a Gaussian function ($\exp(-\Delta z^2/L_z(\omega)^2)$), solid line.

Figure 12. Spanwise coherence function of the top wall pressure obtained by the 3D LES computations ($\theta = 90^\circ$). Data are fit with a Laplacian function ($\exp(-\Delta z/L_z(\omega))$), solid line.

Table 4. Computed spanwise coherence length, $L_z(\omega)$, obtained according to the function (Gaussian or Laplacian) used to fit the calculated coherence function of the wall pressure fluctuations at $\theta = 90^\circ$.

<table>
<thead>
<tr>
<th>$St$</th>
<th>0.19</th>
<th>0.38</th>
<th>0.76</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_z(\omega)$ (Gaussian), $\Gamma_y(\omega) = 0.5$</td>
<td>$7.86 , D$</td>
<td>$0.56 , D$</td>
<td>$0.074 , D$</td>
</tr>
<tr>
<td>$L_z(\omega)$ (Laplacian), $\Gamma_y(\omega) = 0.5$</td>
<td>-</td>
<td>$0.45 , D$</td>
<td>$0.060 , D$</td>
</tr>
</tbody>
</table>
The coherence lengths listed in Table 4 shows that at the vortex shedding frequency $L_c(\omega_0) = 7.86D$ falls within the range $L_s < L_c(\omega) < L_z$ of Kato’s formulation, Eq. (6) and for the other two frequencies $L_c(\omega)$ is much lower than $L_s$, which is in its upper range, $L_c(\omega) \leq L_s$. The results indicate that around the vortex shedding frequency the wall fluctuating pressures are highly correlated up to a spanwise distance of $L_c(\omega_0) = 7.86D$ and weakly correlated for lengths beyond $7.86D$. On the other hand, at other frequencies the fluctuating pressures are assumed to be weakly correlated for span lengths higher than $L_s$. Thus, in this work, at the vortex shedding frequency, using the second expression of Eq. (6), the sound spectrum is corrected adding a value of $\Delta SPL_{corr}(\omega_0) = 15.03\text{dB}$ and at other frequencies, as $L_c(\omega \neq \omega_0) \leq L_s$, the sound is corrected using the first expression of Eq. (6), i.e., the fluctuating wall pressures are assumed to be uncorrelated for lengths beyond $L_z$, which gives $\Delta SPL_{corr}(\omega \neq \omega_0) = 10.05\text{dB}$. The corrected sound spectrum, matching span length used in the experiment of $L = 25.3D$, is finally presented in Fig. 13. Comparing the experimental and the numerical spectra at the vortex shedding frequency, the predicted peak of $SPL$ is 0.5 dB higher than the corresponding value of Revell et al. and the computations give an $OASPL$ value of 100.93 dB which is approximately 1 dB above the experimental result. Therefore, the present computations agree fairly well with the experimental data.

Figure 13. Comparison of sound pressure level spectrum obtained from LES, corrected by Kato et al. method, with the experimental results of Revell et al. Microphone located 128D away from the cylinder axis, positioned 90º from its the stagnation point.

Regarding the second correction method proposed by Seo & Moon, assuming a statistical homogeneity of the wall pressure fluctuations in the spanwise direction and revising the works of Kato et al. and Perot et al., Seo & Moon developed a simple correction method. It assumes that the coherence decay of the far-field fluctuating acoustic pressure in function of the spanwise distance can be expressed by a Gaussian function (assumption based on the works of Casalino & Jacob and Jacob et al.). The main difference from Kato’s correction method is that the coherence functions and consequently the coherence length are calculated based on the far-field acoustic fluctuating pressure obtained from the integral solutions of the wave equation, Eq. (3). Particularly, the approach of taking the obtained far-field sound pressure, dividing the wall into various subsections, for estimating the correlation length was previously proposed by Perot et al. After developing a more complex formulation, Seo & Moon approximated it to a simpler form which is given by the following expression:
This formulation is quite similar to Kato’s correction method, \( L_c(\omega) \) is the correlation length which is obtained from the far-field acoustic fluctuating pressure calculated at the observer position. Depending on the degree of correlation over the cylinder span, represented by \( L_c(\omega) \), three expressions can be used. The first expression, \( 10\log[L/L_c]\ ), assumes (as an approximation) that the fluctuating pressure for spanwise distances beyond \( L_c \) is not correlated, whereas the third expression \( 20\log[L/L_c]\ ) assumes a perfect correlation of the fluctuating pressure along all the cylinder span (\( L \)). The correlation length \( L_c(\omega) \) can be obtained by calculating the acoustic spanwise coherence function \( \gamma_{ij}(\omega) \) which is given by,

\[
\gamma_{ij}(\omega) = \frac{\Re({\hat{p}_i}{\hat{p}_j}^*)}{\sqrt{|{\hat{p}_i}|^2|{\hat{p}_j}|^2}}
\]

where \( {\hat{p}_i} \) and \( {\hat{p}_j} \) are the acoustic pressures (frequency domain) calculated from each wall subsection located at different spanwise positions. In others words, the cylinder wall is equally divided into a certain number of subsections along the cylinder span. The sound radiated from each subsection wall at the observer position (128D away above the cylinder axis) is calculated from the integral solutions of the wave equation, Eq. (3). Therefore, each fluctuating sound pressure, \( {\hat{p}_i} \), can be associated to a cylinder span position which allows to obtain \( \gamma_{ij}(\omega) \) in function of the spanwise distance between \( i \) and \( j \), i.e., \( \Delta z_{ij} \). In the present work, the wall surface of the simulated cylinder was divided equally into 48 subsections along the cylinder span length. Subsequently, the acoustic pressure, \( {\hat{p}_i} \), radiated from each of the 48 subsections was separately calculated using the porous FW-H formulation, Eq. (3), for an observer located 128D away above (\( \theta = 90^\circ \)) the cylinder axis. The same way as done with \( \Gamma_y(\omega) \), the obtained values of the acoustic spanwise coherence function were all ensemble averaged according to the spanwise distance between each wall subsection (\( \Delta z_{ij} \)). Finally, the obtained acoustic coherence function for three Strouhal numbers (\( St = 0.19, 0.38 \) and 0.76) are presented in Fig. 14 (symbols) which is only valid at most up to half the simulated cylinder span (\( \Delta z_{ij} / D < 1.25 \)). In Fig. 14, it is observed that at the Karman shedding frequency the coherence function decays very slowly whereas at the other two higher frequencies the coherence functions drops very rapidly with the increase of the spanwise distance. These results show that around \( St = 0.19 \) the correlation of the fluctuating acoustic pressures decreases very slowly (values close to unity) whereas for higher frequencies, such as \( St = 0.38 \) and \( St = 0.76 \), the acoustic pressure fluctuations are already weakly correlated for spanwise distances higher than half the cylinder diameter. According to the correction method of Seo & Moon\(^7\), Eq. (8), the acoustic correlation length, \( L_c(\omega) \), is determined by the Gaussian function \( \exp(-\Delta z^2/L_c(\omega)^2) \) which best fits the acoustic spanwise coherence function. Therefore, in Fig. 14, the spanwise coherence functions of each of the three frequencies (\( St = 0.19, 0.38 \) and 0.76) are fitted by a Gaussian function (solid line). The coherence lengths (\( L_c(\omega) \)) values obtained from each Gaussian function are presented in Table 5. In Fig. 14, the Gaussian function does not seem to fit so well the coherence function values for \( St = 0.38 \) and \( St = 0.76 \), however the Gaussian function fitted fairly well the corresponding results of Seo & Moon\(^7\) for these two same Strouhal numbers. As Perot \textit{et al}.\(^3\) employed a Laplacian function to fit the corresponding calculated acoustic coherence function and Casalino & Jacob\(^33\) stated that either a Gaussian or a Laplacian function are consistent options, in Fig. 15 the calculated acoustic spanwise coherence function is also fitted with a Laplacian function, \( \exp(-\Delta z/L_c(\omega)) \). Comparing the Gaussian and the Laplacian functions employed to fit to the calculated coherence functions for \( St = 0.38 \) and \( St = 0.76 \) (see Figs. 14 and 15), it can be easily noted that the Laplacian function (Fig. 15) is here a more appropriate choice to represent the decay behavior of the computed coherence function than the Gaussian fitting approach. Table 5 also presents the
coherence length, $L_c(\omega)$, obtained by a Laplacian function fitting for $St = 0.38$ and $St = 0.76$. At $St = 0.19$, the Laplacian function does not fit at all the corresponding calculated coherence function, thus no result of $L_c(\omega)$ is shown in Table 5. Comparing both correlation lengths $L_c(\omega)$ and $L_z(\omega)$, no difference at all is noted for $St = 0.76$ and for $St = 0.38$ the value of $L_c(\omega)$ is slightly higher than $L_z(\omega)$, however this difference would not change the sound correction. If $L_z(\omega)$ were replaced by $L_c(\omega)$, $L_z(\omega)$ would still fall within the upper range ($\sqrt{\pi}L_c(\omega) \leq L_z$) of Seo & Moon\textsuperscript{7} formulation, Eq. (8).

Figure 14. Spanwise coherence function of the fluctuating sound pressure emitted according to each wall subsection. The sound were calculated by the FW-H analogy, Eq. (3), for an observer located 128 $D$ away from the cylinder axis ($\theta = 90^\circ$). Data are fit with a Gaussian function ($\exp(-\Delta z^2/L_c(\omega)^2)$), solid line.

Figure 15. Spanwise coherence function of the sound pressure emitted according to each simulated span. The sound were calculated by the FW-H analogy for an observer located 128 $D$ away from the cylinder axis ($\theta = 90^\circ$). Data are fit with a Laplacian function ($\exp(-\Delta z/L_z(\omega))$), solid line.
The acoustic coherence lengths values of Table 5 show that at the vortex shedding frequency \( L_c(\omega_o) = 10.13D \) falls within the range \( L_z < \sqrt{\pi} L_c(\omega) < L_z \), according to Seo & Moon\(^7\) formulation, Eq. (8), and for the other two frequencies \( L_c(\omega) \) is in the upper range, \( \sqrt{\pi} L_c(\omega) \leq L_z \), of the correction method. Thus, around \( St = 0.19 \) the acoustic fluctuating pressures are assumed to be highly correlated up to \( L_c(\omega_o) = 10.13D \) and weakly correlated for lengths beyond 10.13D. For other frequencies \( \omega \neq \omega_o \), such as \( St = 0.38 \) and 0.76, the acoustic fluctuating pressures are considered to be uncorrelated for lengths higher than the simulated span length, \( L_z \).

At the vortex shedding frequency the sound spectrum is corrected adding a value of \( \Delta SPL_{corr}(\omega_o) = 18.61 \text{dB} \) according to the second expression presented in Eq. (8). At other frequencies, \( \sqrt{\pi} L_c(\omega) \leq L_z \), the sound is corrected by adding a value of \( \Delta SPL_{corr}(\omega \neq \omega_o) = 10.05 \text{dB} \) according to the first expression of Seo & Moon\(^7\) correction method. In fact, \( \Delta SPL_{corr}(\omega \neq \omega_o) \) value is the same as Kato’s method. The final sound spectrum corrected by Seo & Moon\(^7\) method, which matches the span length used in the experiment \( (L = 25.3D) \), is presented in Fig. 16. Comparing the numerical predicted spectrum with the experimental data at the vortex shedding frequency, the peak of SPL is about 4 dB higher than the corresponding value of Revell et al.\(^1\). The OASPL calculation of the corrected spectrum gives a value of 105.4 dB which is approximately 5 dB above the experimental datum (100 dB). Thus, the acoustic spectrum, obtained from the 3D LES simulation, corrected by Seo & Moon\(^7\) method gives a final spectrum whose sound levels are slightly higher than the experimental data.

### Table 5. Acoustic spanwise coherence length computed according to the function (Gaussian or Laplacian) used to fit the calculated acoustic coherence function.

<table>
<thead>
<tr>
<th>St</th>
<th>( L_c(\omega) ) (Gaussian, ( \exp(-\Delta z^2/L_c(\omega)^2) ))</th>
<th>( L_z(\omega) ) (Laplacian, ( \exp(-\Delta z/L_z(\omega)) ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>10.13 ( D )</td>
<td>-</td>
</tr>
<tr>
<td>0.38</td>
<td>0.72 ( D )</td>
<td>1.05 ( D )</td>
</tr>
<tr>
<td>0.76</td>
<td>0.34 ( D )</td>
<td>0.34 ( D )</td>
</tr>
</tbody>
</table>

Figure 16. Comparison of sound pressure level spectrum obtained from the 3D LES model, corrected by Seo and Moon\(^7\) method, with the experimental results of Revell et al.\(^1\). Microphone located 128 \( D \) away from the cylinder axis, positioned 90º from the stagnation point.
V. Conclusion

The acoustic analogy approach was used to compute the sound generated from a low mach number flow around a circular cylinder in the subcritical regime range. In this approach, the noise sources are computed from CFD computations which are used as input data to the wave equation of Ffowcs-Williams & Hawkings\textsuperscript{5}. Here, the noise sources were obtained from unsteady two-dimensional and three-dimensional CFD computations. The most accurate two-dimensional URANS CFD model was the $k-\omega$ SST transition model. The 2D simulation does not provide flow information over the cylinder span which is needed for sound computation. Then, based on 2D flow data, a perfectly correlated flow over the cylinder span length is assumed. As a consequence, a discrete acoustic spectrum at the harmonic frequencies is obtained. In addition, the span length ($L_s$) over which the flow was assumed completely correlated that best matched the overall acoustic level was $L_s = 5.0D$.

The three-dimensional CFD calculations were performed by LES. Due to computational cost limitations, a shorter cylinder span length of $2.5D$ was used for the 3D CFD computations. Regarding the main flow field parameters, the results of the LES model were in good agreement with the ones predicted experimentally. In order to match the cylinder span length used in the CFD simulation ($L_s = 2.5D$) to the length used in the experiment of Revell et al.\textsuperscript{1} ($L = 25.3D$), two acoustic correction methods proposed by Kato et al.\textsuperscript{10} and Seo & Moon\textsuperscript{7} were applied. The 3D noise computation approach provided sound pressure levels all over the spectrum whose values could be directly compared with the experimental sound spectrum for each given frequency.

The computed sound spectrum corrected by Kato et al.\textsuperscript{10} method was found to be in excellent agreement with the corresponding experimental data of Revell et al.\textsuperscript{1} whereas the sound spectrum obtained with Seo & Moon\textsuperscript{7} correction method provided a slight higher SPL value at the vortex shedding frequency and consequently a higher OASPL value. Despite these higher levels of about 5 dB predicted by Seo & Moon\textsuperscript{7} correction method, the acoustic spectrum shape was mostly in accordance with the results of Revell et al.\textsuperscript{1}.

To conclude this study, the three-dimensional approach of calculating the far-field sound from the unsteady flow provided by LES showed to be of predicting the sound spectrum quite accurately for the case of a three-dimensional flow around a cylinder in the subcritical regime. For future studies, LES simulations with longer cylinder span lengths should be carried out in order to have more information of the unsteady flow along the cylinder span which will allow calculating spanwise correlation lengths more accurately. In addition, we intend to investigate the capability of DES turbulence models to compute the flow around the cylinder and its generated sound. Finally, the flow over the cylinder will be computed considering all coupled Navier-Stokes equations (compressible flow), which will allow using off-body integration surfaces so as to account quadrupole noise sources on the generated sound.

Acknowledgments

The authors would like to thank CAPES, FAPESP and EMBRAER from Brazil for their financial support during the course of this research.

References


20 American Institute of Aeronautics and Astronautics