Queueing Models for 4D Trajectory-Based Aircraft Operations in NextGen

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This paper develops a queueing model for aircraft landings at a single runway under trajectory-based flight operations. The situation is expected to arise in the Next Generation Air Transportation System. Aircraft are assigned scheduled times of arrival, which they meet with some stochastic error. It is assumed that errors are small enough for scheduled order to be maintained, and they follow a normal distribution. A recursive queueing model is formulated, and Clark's approximation for the maximum of a finite set of random variables is employed to analytically approximate the mean and variance of flight delays. For a given airport demand and capacity it is shown that total delay is proportional to the standard deviation of the stochastic error; this result facilitates the estimation of the system's total delay through lookup tables. The approximation method is found to be reasonably accurate and very fast computationally.

Nomenclature

\( A \) = actual arrival time at the runway in the absence of queue
\( a \) = scheduled arrival time at the runway in the absence of queue
\( \bar{A} \) = stochastic deviation from scheduled arrival time in the absence of queue
\( D \) = actual landing time at the runway
\( d \) = deterministic landing time at the runway
\( \bar{D} \) = stochastic deviation from deterministic landing time
\( N(\mu,\sigma) \) = normal distribution
\( h \) = headway
\( i \) = flight index
\( \sigma \) = standard deviation of a normal distribution

I. Introduction

The nation's air transportation system (NAS) will incur major transformations in the coming years, developing towards the so-called Next Generation Air Transportation System (NextGen). NextGen features a shift from the current static system of routes and sectors to one that is adaptive to weather, traffic, and user preferences. Users will exchange coordinates information and supply the Air Navigation Service Provider with greater amounts of information about future traffic demand. This will be used to anticipate and resolve conflicts well in advance, reducing the need for tactical air traffic control. It will also allow controlled times of arrival into busy terminals, weather-impacted airspace, and other bottlenecks. This transformation is expected to greatly reduce human operator workload and significantly increase airport and airspace capacity.

The motivation for this research is the fact that the ability to control and predict 4D aircraft trajectories (4DT) with high precision is a cornerstone of NextGen. 4DT capability, with time being the fourth dimension, is defined as the ability to precisely fly an assigned 3D trajectory while meeting specified timing constraints on arrival at waypoints. This will allow high density flows that rely on controlled times of arrival for critical resources, including entry and exit to/from airspaces, taxiways, and runways. When traffic intensity is lower, it will also permit autonomous operations in which aircraft self-separate and may alter their trajectories without obtaining a clearance.

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However, even with the deployment of the very best 4D trajectory precision and navigation tools, adherence to 4D trajectories will not be perfect. Sources of imprecision include airframe-to-airframe variation in aerodynamic performance, limitations in wind prediction capability, variations in flight crew technique, and varying degrees of exactitude in navigational performance. As the NAS evolves from its current state to a future condition where location precision are maximized, a spectrum of trajectory uncertainty will be manifested. That will range from low precision, corresponding to today's operations in the NAS, to high precision, brought on by full deployment of precision navigation and 4DT trajectory awareness tools.

The objective of this research is to model the NAS using queueing theory in a way that accounts for various levels of trajectory uncertainty in all intermediate phases of precision navigation deployment. Existing queueing models typically assume that the arrival process is a non-homogeneous Poisson process. While the Poisson assumption might be reasonable for operations as they are currently taking place in the NAS, it is not a good way to study the propagation of uncertainty under NextGen concepts, such as trajectory-based operations, designed to reduce uncertainty, because no controls are allowed for the reduction of this variance. For more realistic levels of trajectory uncertainty, and for the reduced levels expected as a benefit of NextGen technologies, a better recourse is schedule-based analysis.

Thus, a queueing model with scheduled arrivals is proposed in this paper, to analyze flight delays in a high trajectory precision operational environment, as currently being planned for NextGen. Within transportation engineering context, queueing models with scheduled arrivals have been proposed. In Ref. 3, single server queueing systems are examined, where customers arrive according to a schedule but not punctually, and must be served in the order of the schedule. Analytical expressions are derived for the case where successive customers are not likely to interfere with each other. In the present paper though, interferences are allowed and approximate analytical formulations are developed, while maintaining the assumption of FSFS service.

II. The Model and an Approximate Solution

A. Model Formulation

For a given schedule of flight arrivals at a destination airport, it is assumed that aircraft are instructed to fly 4D trajectories in order to arrive at the airport just in time to make these times. Our queueing model includes a single server, which is the airport's runway, and a series of airplanes that want to land. Each plane \( i \) has an actual arrival time at the server \( A_i \) that consists of a deterministic and a stochastic portion. The deterministic component \( a_i \) is the scheduled arrival time at the runway. However, due to imprecision to trajectory adherence, there is some stochastic deviation from the scheduled arrival time that is denoted as \( \tilde{A}_i \). Therefore, we have \( A_i = a_i + \tilde{A}_i \).

Central to our model development is the assumption that deviations \( \tilde{A}_i \)'s are small enough, therefore aircrafts are served on a First-Scheduled-First-Served basis. In this way, the actual time airplane \( i \) departs from the server (i.e lands on the runway), \( D_i \), would be either \( A_i \) if there were no queue at the server by the time it arrived, or the time the previous aircraft left plus the minimum required separation headway \( h_{i-1,i} \):

\[
D_i = A_i \\
D_i = \max\left(A_i, D_{i-1} + h_{i-1,i}\right), \quad \forall i \geq 2
\]

If there were no stochasticity in the system, the deterministic time of departure from the server would be:

\[
d_i = \max\left(a_i, d_{i-1} + h_{i-1,i}\right), \quad \forall i \geq 2
\]

Accounting for stochasticity, the departure time from the server of airplane \( i \) is:

\[
D_i = d_i + \tilde{D}_i
\]

The distribution of the stochastic component \( \tilde{D}_i \) clearly depends on \( \tilde{A}_i \), which captures all stochastic effects that cause flight \( i \) to arrive at a time other than its scheduled one \( a_i \):

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The mean random variables. If we let result in Ref. 4 normal rand B.
aircraft aircraft with and without 4DT’s capability. This way one could assume two different values for the standard deviation, without 4DT capability will be relegated to "classic airspace" in which operations are much the same as today. Priority they receive. Aircraft with such capability will have access to traje and communicate precise 4DT trajectories will be a key criterion determining where capabilities of adherence to 4D trajectories. For example, in NextGen as currently envisioned, the capability to fly
deviations flight crew technique, and varying degrees of exactitude in navi and variance
already been incorporated in the estimation of scheduled arrival times weather or en route congestion that cause significant amounts of delays; lateness effects due to such factors have already been incorporated in the estimation of scheduled arrival times . The sources of imprecision include airframe-to-airframe variation in aerodynamic performance, limitations in wind prediction capability, variations in flight crew technique, and varying degrees of exactitude in navigational performance, Therefore schedule deviations are the sum of independent stochastic factors, and for this reason they are assumed to be normally distributed.

Finally, values for schedule deviation could be aggregated to represent classes of aircraft that have similar capabilities of adherence to 4D trajectories. For example, in NextGen as currently envisioned, the capability to fly and communicate precise 4DT trajectories will be a key criterion determining where aircraft can fly and what priority they receive. Aircraft with such capability will have access to trajectory-based airspace, while aircraft without 4DT capability will be relegated to "classic airspace" in which operations are much the same as today. In this way one could assume two different values for the standard deviation, and , in order to roughly represent aircraft with and without 4DT's capability. Moreover, even within the same equipment class, one could differentiate aircraft’s ability to precisely fly 4D trajectories according to the en-route weather conditions they encounter.

B. Clark’s Approximation Method

In equation (1), for both terms of the operator are normally distributed. The max operation on normal variables, contrary to the add operation, does not yield a normal random variable. A well-known result in Ref. 4 derives analytical formulas for the mean and variance of the maximum of two normally distributed random variables. If we let and to be normally distributed random variables, and , and to represent the correlation coefficient between and , we then define . The mean and variance of are expressed analytically as follows.

\[
\mu_Z = \mu_x \Phi(\alpha) + \mu_y \Phi(-\alpha) + \gamma \varphi(\alpha) \\
\sigma^2_Z = \left(\sigma^2_x + \mu^2_x\right) \Phi(\alpha) + \left(\sigma^2_y + \mu^2_y\right) \Phi(-\alpha) + \left(\mu_x + \mu_y\right) \gamma \varphi(\alpha) - \mu^2_Z
\]

where

\[
\gamma \equiv \left(\sigma^2_x + \sigma^2_y - 2 \rho \sigma_x \sigma_y\right)^{1/2} \\
\alpha \equiv \frac{\mu_x - \mu_y}{\gamma} \\
\varphi(x) \equiv \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \\
\Phi(y) \equiv \int_{-\infty}^{y} \varphi(x) \, dx
\]

The above formulas give the exact mean and variance of . The approximation is introduced by assuming that follows a normal distribution with mean and variance .

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In the context of our problem with scheduled aircraft arrivals, Clark's method can be used recursively for $i \geq 2$ to approximate $D_i$'s as normal random variables, and estimate their mean $E(D_i)$ and variance $\text{Var}(D_i)$ for all $i$'s:

$$E(D_i) = a_i \Phi(\alpha_i) + \left[ E(D_{i-1}) + h_{i-1} \right] \Phi(-\alpha_i) + \gamma_i \varphi(\alpha_i)$$  \hspace{1cm} (2)

$$\text{Var}(D_i) = \left( \sigma_i^2 + a_i^2 \right) \Phi(\alpha_i) + \left[ \text{Var}(D_{i-1}) + \left[ E(D_{i-1}) + h_{i-1} \right]^2 \right] \Phi(-\alpha_i)$$

$$+ \left[ a_i + E(D_{i-1}) + h_{i-1} \right] \gamma_i \varphi(\alpha_i) - \left[ E(D_i) \right]^2$$ \hspace{1cm} (3)

where

$$\gamma_i = \left( \sigma_i^2 + \text{Var}(D_{i-1}) \right)^{1/2}$$ \hspace{1cm} (4)

$$\alpha_i = \frac{a_i - E(D_{i-1}) - h_{i-1}}{\gamma_i}$$ \hspace{1cm} (5)

Equations (2)-(5) are easy to program and they are computationally efficient. Finally, for a stream of $n$ flights scheduled for landing, the total expected delay is defined as:

$$\bar{W}_n = \left[ \sum_{i=1}^{n} E(D_i) - a_i \right]$$ \hspace{1cm} (6)

This completes the formulation of our queueing model. In summary, the model requires as inputs a schedule of arrival times $a_i$, a capacity profile expressed in terms of time separation headways $h_{i-1}$, and a set of trajectory adherence errors $\sigma_i$. These, coupled with the assumption for relatively small $\sigma_i$'s, enable the estimation of expected flight delays through Clark's approximation method.

**C. Approximation Error**

Although the maximum $Z$ of two normal random variables $X$ and $Y$ is not normally distributed, our analysis is based on approximating $Z$ with a normal random variable. In particular, in estimating $D_i = \max \{ A_i, D_{i-1} + h_{i-1} \}$ it is assumed that $D_{i-1}$ is normally distributed. That enables the estimation of the mean and variance of $D_i$, which is then also approximated as a normal random variable. Each pair-wise operation introduces some error that is propagated and might affect the accuracy of our estimates. The error should depend on the relative distance $\alpha$ between the means of $X$ and $Y$. For a thorough analysis on this topic, see Ref. 5.

In this paper, we are interested in the approximation error when estimating the expected total delay $\bar{W}_n$ for a sequence of $n$ aircraft arrivals. An experiment using empirical data from SFO was performed in

![Figure 1. Comparison of analytical approximation and simulation results for expected total delay $\bar{W}_n$](image-url)
order to test the amount of approximation error. A typical daily schedule and a capacity scenario at SFO were employed as inputs to our queueing model. The adherence error $\sigma$ was assumed homogeneous for all aircraft. We then compared the analytical model estimate for $\hat{W}_a$ with that obtained from a Monte Carlo simulation, which is considered as ground truth. The experiment was repeated for different values of $\sigma$, and the results are plotted in Fig. 1.

It is clear that the error (measured as the vertical difference between the analytical and simulation data points) is small, on the order of $\pm 3\%$. Cases with heterogeneous errors still need to be explored though; some preliminary results, not presented here, also show that the amount of error is small.

III. Metering Application

Here we present a model for queueing systems with metered arrivals, where the term metered implies a uniform time separation between successive arrivals. This situation can correspond to an airport where average demand over a time period over exceeds capacity. In addition, the model is applicable in cases when the only information available is the total number of flights scheduled to land, but not a detailed timetable.

It is important to clarify which component of delay the metering model calculates. When metering is implemented, each aircraft is assigned a landing time slot, typically no earlier than its initial (i.e. before metering was activated) scheduled arrival time. The difference between the assigned arrival slot and the scheduled time of arrival is referred here as deterministic delay, because it can be planned in advance and taken on the ground. Its computation is straightforward and, thus, it is not the focus of this section. In addition to the deterministic delay, the aircraft might incur “stochastic” delay that is caused by imprecise adherence in arriving at its assigned slot time. In the following discussion, we estimate this stochastic delay due to imperfect execution of assigned 4D trajectories.

We focus on the single airport problem, and make the following assumptions:

1) Separation requirements are the same between airplanes and across time, i.e. the required minimum time headway $h$ between aircraft landings is constant.
2) Flight arrivals are uniformly scheduled during the analysis period, with time separation headway $\alpha$.
3) Standard deviation has a uniform value $\sigma$ across all random variables $\hat{A}_i$.

Typically aircraft are metered to an airport at a rate less than or equal to the runway’s capacity for landings, $a \geq h$. In a fully deterministic environment, there should be no delays, since the rate at which airplanes arrive at the server is smaller than the rate at which aircraft can depart from the server. However, if an airplane arrives later than its scheduled arrival time, then it might cause delay to the airplanes behind it waiting to enter service. We are interested in quantifying the expected amount of that delay, and for that it suffices to estimate the mean of $\tilde{D}_i$. Since $a \geq h$, the deterministic landing times would be $d_i = ia$ and Eq. (1) becomes:

$$\tilde{D}_i = \hat{A}_i$$  \hspace{1cm} (7a)

$$\tilde{D}_i = \max\{\hat{A}_i, \tilde{D}_{i-1} + h - a\}, \ \forall i \geq 2$$  \hspace{1cm} (7b)

To estimate the mean and variance of $\tilde{D}_i$ we employ Clark’s approximation method:

$$E(\tilde{D}_i) = E(\tilde{D}_{i-1}) + h - a \Phi(-\alpha_i) + \gamma_i \Phi(\alpha_i)$$  \hspace{1cm} (8)

$$\text{Var}(\tilde{D}_i) = \sigma^2 \Phi(\alpha_i) + \left[ \text{Var}(\tilde{D}_{i-1}) + \left[ E(\tilde{D}_{i-1}) + h - a \right]^2 \right] \Phi(-\alpha_i)$$

$$+ \left[ E(\tilde{D}_{i-1}) + h - a \right] \gamma_i \Phi(\alpha_i) - \left( E(\tilde{D}_i) \right)^2$$  \hspace{1cm} (9)

where

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\[ \alpha_i = \frac{-E(\tilde{D}_{i-1}) - h + a}{\gamma_i} \]  

(10)

\[ \gamma_i = \left(\sigma^2 + \text{Var}(\tilde{D}_{i-1})\right)^{1/2} \]  

(11)

Next, we are interested in deriving an analytical expression for \( E(\tilde{D}_i) \) and \( \text{Var}(\tilde{D}_i) \), i.e. a formulation that would not be recursive. However, this is a rather difficult task. Instead, we employ the following proposition:

**Proposition 1.** Controlling for the dimensionless constant \( \Delta \triangleq (a - h) / \sigma = \delta \), the mean and standard deviation of \( \tilde{D}_i \) are proportional to \( \sigma \):

\[ E(\tilde{D}_i) = m_i \sigma \]  

(12)

\[ \text{Var}(\tilde{D}_i) = v_i \sigma^2 \]  

(13)

where \( m_i \) and \( v_i \) are parameters to be estimated.

**Proof.** If \( E(\tilde{D}_{i-1}) = m_{i-1} \sigma \) and \( \text{Var}(\tilde{D}_{i-1}) = v_{i-1} \sigma^2 \), then Eq. (11) gives \( \gamma_i = \beta_i \sigma \), where \( \beta_i = (1 + v_{i-1})^{1/2} \). Since \( (a - h) = \delta \sigma \), from Eq. (10) we note that \( \alpha_i \) is independent of \( \sigma \). Therefore Eq. (8) becomes:

\[ E(\tilde{D}_i) = \left( m_{i-1} \sigma + \delta \sigma \right) \Phi(-\alpha_i) + \beta_i \sigma \varphi(\alpha_i) \]

or

\[ E(\tilde{D}_i) = m_i \sigma \]

Now Eq. (9) becomes:

\[ \text{Var}(\tilde{D}_i) = \sigma^2 \Phi(\alpha_i) + \left( v_{i-1} \sigma^2 + (m_{i-1} \sigma + \delta \sigma)^2 \right) \Phi(-\alpha_i) \]

\[ + (m_{i-1} + \delta) \beta_i \sigma^2 \varphi(\alpha_i) - m_i^2 \sigma^2 \]

or

\[ \text{Var}(\tilde{D}_i) = v_i \sigma^2 \]

We observe that for \( i = 2 \):

\[ E(\tilde{D}_2) = \delta \sigma \Phi(-\delta \sqrt{2} / 2) + \sigma \sqrt{2} \varphi(\delta \sqrt{2} / 2) = m_2 \sigma \]

\[ \text{Var}(\tilde{D}_2) = \sigma^2 \Phi(\delta \sqrt{2} / 2) + \left( \sigma^2 + \delta^2 \sigma^2 \right) \Phi(-\delta \sqrt{2} / 2) + \delta \sigma^2 \sqrt{2} \varphi(\delta \sqrt{2} / 2) = v_2 \sigma^2 \]

Hence, for \( i \geq 3 \) the result follows by induction. \( \therefore \)
For $\sigma = 1$, $\Delta = a-h$ and we define $\tilde{Z}_i = \max(\tilde{A}_i, \tilde{Z}_{i-1})$, where in this case $\tilde{A}_i \sim N(0,1)$. Therefore, it suffices to consider cases for $a-h$ and use Eq. (8)-(11) to recursively compute the values for $m_i = E(\tilde{Z}_i)$ and $\nu_i = Var(\tilde{Z}_i)$. Figure 2 depicts curves for $E(\tilde{Z}_i)$ for several values of $a-h$.

Thus for any value of $\sigma$, $\alpha$ and $h$, values for $E(\tilde{D}_i)$ are computed by selecting the appropriate curve from Fig. 2 and multiplying $E(\tilde{Z}_i)$ by $\sigma$.

![Figure 2. Expected delay $E(\tilde{Z}_i)$ for several values of $\Delta$.](image)

In this way, instead of an analytical formula, values for $E(\tilde{D}_i)$ can be estimated through a lookup table and with the same level of accuracy.

It is notable that the system reaches a steady state, in most cases after the first ten customers, and thus each flight incurs the same amount of expected delay $E(\tilde{Z}_i)\sigma$. The only exception is when $a = h$, which is further analyzed in section A.

Finally, for a stream of $n$ flights scheduled for landing and a given value $a-h$, the total expected delay is equal to:

$$W_n = \left[\sum_{i=1}^{n} E(D_i - \alpha a)\right] = \left[\sum_{i=1}^{n} E(\tilde{Z}_i)\right] \sigma \quad (14)$$

Therefore, the expected total delay of the system increases linearly with the standard deviation $\sigma$ of the stochastic error $\tilde{A}_i$.

\section*{A. Case when $a=h$}

In this section we focus in the case where there is no buffer time in the metered arrival schedule, and the metered time spacing $\alpha$ between consecutive arrivals is equal to the minimum required headway $h$. As observed in Fig. 2...
that results in the highest delays, which is expected since deviation from metered arrival time cannot be absorbed in the schedule. Also, a steady state is not attained for the system in this case.

Interestingly, the problem is equivalent with finding the maximum of \( n \) independent and identically distributed normal random variables. When \( a = h \), Eq. (7b) reduces to 
\[
\bar{D}_i = \max\left(\bar{A}_i, \bar{D}_{i-1}\right), \quad \forall i \geq 2;
\]
thus for \( i = 3 \)
\[
\bar{D}_3 = \max\left(\bar{A}_3, \max\left(\bar{A}_1, \bar{A}_2\right)\right).
\]
Therefore
\[
\bar{D}_n = \max\left(\bar{A}_1, \cdots, \bar{A}_n\right) \cdot \sigma,
\]
where \( \bar{A}_i \sim N(0,1) \quad \forall i \geq 1 \) (15)

An analytical solution as well as numerical tables for this problem can be found in Ref. 6. Comparing the exact results with the approximation values listed in Table A1 and depicted in Fig. 2, one could observe a rather small error in the approximation.

B. Numerical Example

A numerical example is provided in this section. Let's assume that the Airport Acceptance Rate (AAR) at an airport \( B \) is 40 arrivals/hour, which corresponds to a minimum time headway between consecutive arrivals of \( h = 90 \) sec. Two scenarios are then considered: a) flights bounded to \( B \) are metered at a rate \( a = 100 \) sec, and b) flights bounded to \( B \) are metered at a rate \( a = 90 \) sec. In addition, aircraft are assigned to fly 4D trajectories, to which they can adhere with a precision \( \sigma = 20 \) sec. Therefore, \( \Delta = 0.5 \) in the first case, while \( \Delta = 0 \) in the second case. If the demand drops significantly (i.e. to zero) after one hour, we want to estimate the total expected delay for these 40 flights that are metered to airport \( B \).

The results are presented in Table 1 in the Appendix. The total expected delay is 386 sec when \( \Delta = 0.5 \), and 1373 sec when \( \Delta = 0 \). The latter is approximately three times higher than in the first case. Note that simply reading the graph in Fig. 2 could have drawn this conclusion on the order of magnitude of delays. While the deterministic total delay decreases with the buffer \( a - h \), the system incurs higher stochastic delay due to its reduced capacity for absorbing stochastic deviations from schedule.

C. Approximation Error

The values of \( E\left(\bar{Z}_i\right) \) depicted in Fig.2 have been computed using Clark’s approximation method. Therefore, the accuracy of the estimations must be investigated. We compare the estimations for \( E\left(\bar{D}_i\right) \) computed through the approximation method with those obtained through a full Monte Carlo simulation consisting of \( 10^5 \) runs. In the simulation experiment we assumed \( \sigma = 20 \) sec, which is an expected figure for 4D operations in NextGen. In other words, the curves in Fig. 2 were scaled by a factor of 20 and then compared to those obtained from a simulation. As a metric of comparison, the percent difference of approximate and simulation results was calculated:

\[
ed(\%) = \frac{E(\bar{D}_i)^{approx} - E(\bar{D}_i)^{sim}}{E(\bar{D}_i)^{sim}} \cdot 100
\]
Several scenarios for $\Delta$ were considered and values for $E(\bar{D})$ were estimated. The magnitude of errors $\epsilon$ was inserted in a contour plot, which is illustrated in Fig. 3.

![Contour plot](image)

**Figure 3. Contour plot of percent errors $\epsilon$ in comparing approximate and simulation results for $E(\bar{D})$, when $\sigma = 20$ sec**

It is clear that for small values of $\Delta$ the approximation error in estimating $E(\bar{D})$ is small, ranging between -5% and 1%. Higher errors, ranging between -6% and -12%, are observed when $\Delta$ becomes greater than 0.8; however, according to Fig. 2, for such $\Delta$ the values of $E(\bar{D})$ are very low and the approximation error $\epsilon$ is then subject to simulation noise.

### IV. Conclusion

In this paper a queueing model for aircraft landings at a single runway under trajectory-based flight operations is employed. Aircraft are assigned 4D trajectories and landing times at an airport, which they meet with some stochastic error. It was assumed that the error is small enough for scheduled order of landings to be maintained, and that it follows a normal distribution. A recursive queueing model was formulated, and Clark's approximation for the maximum of a finite set of random variables was employed to analytically approximate the mean and variance of flight delays. The model is computationally efficient, while its accuracy was tested with empirical data and was found to be on the order of ±3%. Next, a situation where flights are metered at a constant rate to an airport was examined. For a given $\Delta$ (buffer between arrivals scaled by the adherence error) it was shown that total delay is proportional to the adherence error $\sigma$; this result facilitates the estimation of the system's total delay through lookup tables.

Extending this research will allow for greater adherence errors that would result in re-sequencing the order in which flights are landed. In addition, assumptions on constant headway $h$ and uniform adherence error $\sigma$ can be relaxed. Finally, the predictions of the model could be compared with delay forecasts from a stochastic model that assumes Poisson arrivals, and from a deterministic model that assumes perfect execution of assigned 4D trajectories. The latter two models correspond to the two endpoints of trajectory uncertainty for operations in NextGen, and are analyzed in Ref. 7.
Appendix

Table 1. Results of numerical example

<table>
<thead>
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<th>Customer Index</th>
<th>$\Delta = 0.5$</th>
<th>$\Delta = 0$</th>
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<tbody>
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<td>$i_a$ (sec) $E(D_i)$ (sec)</td>
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Total (sec) 386 1373
Acknowledgments

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References