Reconfigurable Control Allocation Technology Using Weighted Least Squares for Nonlinear System in Unmanned Aerial Vehicle

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Reconfigurable control allocation research is important to multidisciplinary science and engineering applications. In particular, the proposed research plays a significant role in enhancing the safety, reliability and fault tolerance capability of Unmanned Aerial Vehicle (UAV), which is one of the most active research and development areas. The main objective of this paper is to introduce and evaluate UAV reconfigurable control system design against control surfaces faults without modifying the baseline controller. The faults introduced are in the form of partial loss and stuck at unknown position on the UAV control surfaces. Weighted Least Squares (WLS) control reallocation algorithm with application to UAV was investigated. The paper is undertaken in a nonlinear UAV model ALTAV (Almost-Light-Than-Air-Vehicles), developed by Quanser incorporation. Different faults have been introduced in control surfaces with different trajectory commanded inputs. Gaussian noise was introduced in the model. Comparisons were made under normal situation, the case without control reallocation, and the case with control reallocation method. Simulation results show the satisfactory reconfigurable flight control system performance using the WLS control reallocation method for ALTAV nonlinear UAV benchmark.

Nomenclature

\[ M = \text{Mass of the vehicle} \]
\[ x = \text{Vehicle } x \text{ position} \]
\[ y = \text{Vehicle } y \text{ Position} \]
\[ z = \text{Vehicle } z \text{ Position} \]
\[ \theta = \text{Euler angle} \]
\[ \gamma = \text{Euler angle} \]
\[ \phi = \text{Euler angle} \]

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\[ \gamma \] = Euler angle
\[ J_\phi \] = Moments of inertia about \( x \) axis
\[ J_\gamma \] = Moments of inertia about \( y \) axis
\[ J_\theta \] = Moments of inertia about \( z \) axis
\[ F_B \] = Buoyant force resulting from the volume of helium in the vehicle
\[ F_i \] = Force magnitude of \( i \)th \((i = 1, \ldots, 4)\) motor
\[ l \] = Perpendicular distance between the motors and vehicle center of gravity
\[ C_i \] = This is the drag coefficient in the direction \( i \in [x, y, z, \theta, \gamma, \phi] \) which serves as a damping term for the motor in that direction
\[ \rho \] = Angular offset from the vertical axis of the motor thrust vectors

\[ \text{I. Introduction} \]

UAV is an aircraft that is driven by power, flying without an on-board operator and can be re-usable. One of the most appealing topics among the control research community is the application of modem control theory to unmanned aerial vehicles [1]. Such vehicles can be controlled remotely by an operator on the ground, or autonomously via a pre-designed program. This interest is due to their wide-range field of applications in both civil and military. Applications like traffic surveillance, area mapping, forest fire detection, require high maneuverability of the aircraft and robustness of the control algorithm with respect to parameter uncertainty and disturbances like wind and other weather conditions. UAV is playing an increasingly important role in the modern high technology benefited from easy of use and several unique characteristics [6]. However, due to the lack of a pilot, UAV loses the ability of making a smart decision as a human pilot can do during an emergency condition. It leads easily to mission failure when UAV fails to work on such an abnormal condition, such as flight computer failure, airborne sensor failure and control surface damage, etc. On the other hand, UAV in battle application demands it having good performance to escape from the opponent’s attack. From domestic and foreign high reliability flight control system development, UAV flight control system requires with high reliability, high survivability to maximally ensure safety of the UAV and equipment, in order for the UAV safely complete reconnaissance and surveillance mission. As the same as UAV, all modern airplanes with pilot depend upon their flight control systems to provide the handling qualities necessary for successful flight. Therefore it is very necessary to develop the flight control system that can enable aircraft successful complete mission in reliable flight under non-fatal fault cases flight control system can solve this problem. Some previous research works were introduced in [4-5], [7-11], [25].

Fault tolerant control system (FTCS) is a system that possesses the ability to adapt the system component failures automatically and capable of maintaining the system stability and satisfied performance despite occurrence of faults. For an overall picture of the FTCS approaches and the application fields, the authors can refer to the bibliographical review of Zhang and Jiang [23]. Its goal is to maintain/enhance safety and reliability with the modern technology. Redundancy is the key for any FTCS to achieve fault tolerant capability. Typically, a FTCS consists of three parts: a reconfigurable control allocation, a Fault Detection and Identification (FDI) scheme, and a control law reconfigurable mechanism. This paper will focus on the development of reconfigurable control allocation. Generally speaking, rely on the information from the fault detection and diagnosis, reconfigurable control allocation can be classified into two categories: 1) Active FTCS includes an on-line FDI (Fault Detection and Identification) scheme and fault handling is carried out based on information on faults delivered by the FDI [2, 12-15, 28]. Various Books on active FTC have also been published recently such as Blanke et als, Noura et als [26, 27]. 2) Passive FTCS is so designed that they are robust to handle certain fault without the need of on-line fault information [16-18].

This paper investigates the Weighted Least Squares method to reconfigurate control system when actuators/effectors have partial loss or stuck at an unknown position. WLS algorithm based on classical active set methods [20 21], finds the optimal solution of the control allocation.

This paper is organized as follows: In Section II, the basic fault modeling is introduced. In Section III, control (re)allocation problem formulation is presented. Weighted Least Squares control re-allocation algorithm is also introduced in the same section. ALTA V (Almost-Light-Than-Air-Vehicle) UAV non-linear model is introduced in Section IV. To illustrate the proposed algorithm, a simulation example, the ALTA V UAV non-linear simulation results is given in Section V. Finally, some concluding remarks and future work are made in Section VI.
II. Fault Modeling

Fault modeling is needed to design and evaluate the performance of the reconfigurable controller. In our approach, the faults considered on control surfaces are

-- Partial loss.
-- Stuck at unknown/zero position

When a loss occurs in a control surface, the actuator would not be able to track the control command given by the baseline controller completely. Therefore a reconfiguration must be done.

A. Partial Loss

One way to quantify the magnitude on the actuator fault is by defining a parameter known as control effectiveness factor $\gamma^i$ (Zhang and Jiang, 2002) [2], $i = 1, 2, \ldots, m$. The control effectiveness factor will represent the loss of a one-to-one relationship between the control command (controller output) and the true actuator action, see a Fig. 1.

The system with the actuator faults modeled by control effectiveness factors can be written as:

$$\dot{x} = Ax + B_f u$$
$$y = Cx$$

where $B_f$, the post-fault input matrix can be modeled as:

$$B_f = B \times \Gamma$$

B is the original control input matrix.

$$\Gamma = \begin{bmatrix}
\gamma^1 & 0 & 0 & 0 \\
0 & \gamma^2 & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \gamma^m
\end{bmatrix}$$

In the matrix $\Gamma$, the $\gamma^i = 1$, $i = 1 \ldots m$ corresponds to the fault-free actuator. $\gamma^i = 0$ corresponds to the situation when the control surface is totally loss. $0 < \gamma^i < 1$ represents partial loss in control effectiveness.

B. Stuck Failure

Let the linearized dynamics of the normal aircraft at a trim condition be:

$$\dot{x} = Ax + Bu$$

If one of the control surfaces is suddenly stuck at an unknown position $w$, then above equation is rewritten as:

$$\dot{x} = Ax + B_f u_f + d_x$$

where $u_f \in \mathbb{R}^{m-1}$ is the remaining control surfaces; $B_f$ the post-failure control influence matrix; $d_x = b_w w$ the input to the aircraft caused by the stuck surface at a position $w$.

This results in change in the control effectiveness matrix, hence stuck actuators reduce the number of healthy
control surfaces. Their effects can be viewed as additional constant disturbances imposed onto the system, which may drive the system away from the desired path. The closed-loop system stability may also be affected due to the loss of some control channels.

III. Control (Re)-allocation Problem Formulation

A. Control Allocation
Control allocation is useful for control of over-actuated systems; control allocation is to distribute the deflections of multiple control surfaces of the aircraft to generate the required control signals, including heading, pitch, roll moments, and forces. Control allocation problem studied following the work of Durham [22].

Fig. 2 is control system structure when control allocation is used. The control system is made up by a control law, specifying which total control effect, \( v \) should be produced and a control allocation, which distributes the desired control demand among the individual actuators, \( u \). In this system, actuators generate a total control effect, \( v_{sys} \), which determines the system behavior. If the control allocation is successful, \( v_{sys} = v \). Today, control allocation is attracting more research interesting in aerospace control, marine vessel control and UAV control [4].

B. Control Re-allocation
Control reallocation is to re-distribute the control signals for cancelling the effects of the faults by using the remaining healthy actuator/effectors in the presence of actuator faults. Fig. 3 shows that control re-allocation is adding into the flight control system.

The algorithm inside the control re-allocation block re-distributes the control signals to the control surfaces and makes the system to achieve acceptable performance. Control allocation block and control reallocation block are placed between the control law and actuators. Its advantages are:
1) Actuator constraints can be taken into account.
2) Reconfiguration can be performed if the effectiveness of the actuators changes over time, or in the event of an actuator failure, without having to redesign the control law.
3) Actuator utilization can be treated independently and can be optimized for the application considered.

The main idea to control reallocation is that once one or more control surfaces get stuck or get partially lost during the flight, control reallocation methods should be able to use the redundancy of operable control surfaces to cancel the effects of the jammed and partial loss of the surfaces and provide the same or almost the same desired control inputs.

Let us now survey the most common methods for control reallocation appeared in the literature.

C. Optimization Based Control Allocation
Optimization based methods rely on the following pragmatic interpretation of the control allocation problem.
Given a virtual control command \( v \), determine a feasible control input \( u \) such that \( Bu = v \). This can be considered in the following way:

If there are several solutions, pick the best one. If there is no solution, determine \( u \) such that \( Bu \) approximates \( v \) as close as possible.

**Description of Method**

As a measure of how “good” a solution or an approximation is, the \( l^p \) norm is used [19, 24]. For a particular \( p \), we will refer to this as \( l^p \)-optimal control allocation. The \( l^p \)-norm of a vector \( u \in \mathbb{R}^m \) is defined as:

\[
\|u\|_p = \left( \sum_{i=1}^{m} |u_i|^p \right)^{1/p}
\]

for \( 1 \leq m \leq \infty \)

The optimal control input is given by the solution to a two-step optimization problem with a choice of \( p=2 \).

\[
u = \arg \min_{u \in \Omega} \|W_u (u - u_d)\|_2 \tag{7a}
\]

\[
\Omega = \arg \min_{u_{\text{min}} \leq u \leq u_{\text{max}}} \|W_v (Bu - v)\|_2 \tag{7b}
\]

\( u_d \) is the desired control input \( W_u \) and \( W_v \) are weighting matrices. Above equation should be interpreted as follows: Given \( \Omega \), the set of feasible control inputs that minimizes \( B u - v \) (weighted by \( W_v \)), pick the control input that \( u - u_d \) (weighted by \( W_u \)). A common technique is to approximately reformulate the sequential optimization problem Eq. (7a) and Eq. (7b) as a weighted optimization problem,

\[
u = \arg \min_{u \in \Omega} \|W_u (u - u_d)\|_2 + \gamma \|W_v (Bu - v)\|_2 \tag{8}
\]

where \( \gamma \gg 1 \), \( B u - v \) should be minimized.

**D. Weighted Least Squares Technique**

In this section, the Weighted Least Squares algorithm is introduced. WLS is a use of the active set method to solve the \( l_2 \) optimal control allocation problem. Based on O. Härkegård [19], active set method is widely used to solver for constrained quadratic programming, and has been proved that optimal solution can be obtained in a finite number of iterations. The use of active set method has two obvious advantages:

1) Reduce the constraints of question, thus enable to solver the question easy.
2) Reduce the possibility of incompatibility to QP sub-problems.

The bound and equality constrained least squares problem may write as follow:

\[
\begin{align*}
\min_u &\|Au - b\|_2 \\
Bu &= v \\
Cu &\geq U
\end{align*}
\]

Here \( C = \begin{pmatrix} I & -I \end{pmatrix} \) and \( U = \begin{pmatrix} \frac{u}{u} \end{pmatrix} \), so \( Cu \geq U \) is equivalent to \( u \leq U \).

The active set method solves this problem by solving a series of equality constraints problems. The idea is that in each step some of the inequality constraints are regarded as equality constrains, and form the working set \( W \), while the remaining inequality constraints are disregarded. The active set of the solution is the working set at the optimum.
When the control allocator is initiated, and there is no previous solution available, \( u^0 = \frac{(u + u)}{2} \) and \( W = 0 \) are selected.

Cost function rewrites as standard form:

\[
\|W_u(u - u_d)\|_2^2 + \gamma \|W_v(Bu - v)\|_2^2 = \left\| \frac{1}{r^2}W_vB \right\|_2 u - \left( \frac{1}{r^2}W_vB \right) \|W_uu_d\|_2^2
\]

\[
A = \left( \begin{array}{c}
\gamma W_vB \\
W_u
\end{array} \right), \quad b = \left( \begin{array}{c}
\gamma W_vv \\
W_uu_d
\end{array} \right)
\]

Solve

\[
u = \arg\min_u \|Au - b\| \quad \text{\( Bu = v \)} \quad \text{\( Cu \geq U \)}
\]

Let \( u^0 \) be a feasible starting point. A point is feasible if it satisfies Eq. (9b) and Eq. (9c). Let the working set \( W \) contain (a subset of) the active inequality constraints at \( u^0 \).

Given a sub-optimal iterate \( u^i \) \( i = 1, 2, \ldots \), find the optimal perturbation \( p \) considering the inequality constraints in the working set as equality constraints and disregarding the remaining inequality constraints. Solve

\[
\min_p \|A(u^i + p) - b\| \quad \text{\( Bp = 0 \)}
\]

\[
p_j = 0, i \in W
\]

For one situation, if \( u^i + p \) is feasible, set \( u^{i+1} = u^i + p \) and compute the Lagrange multipliers in the following form,

\[
A^T(Au - b) = \begin{pmatrix} B^T & C^T_0 \end{pmatrix} \begin{pmatrix} \mu \\ \lambda \end{pmatrix}
\]

If all \( \lambda \geq 0 \), \( u^{i+1} \) is the optimal solution to Eq. (9b). Iteration will stop with \( u = u^{i+1} \), else, remove the constraints associated with the most negative \( \lambda \) from the working set.

For another situation, if \( u^i + p \) is in feasible, there is a need to determine the maximum step \( \alpha \) length such that \( u^{i+1} = u^i + \alpha p \) is feasible. Then add the bounding constraint at \( u^{i+1} \) to the working set.

### IV. Implementation of ALTAV UAV Model

In this paper, the Quanser Almost Lighter than Air Vehicle (ALTAV) UAV model is used in the simulation studies (in real world experiments). They use buoyancy to float in the air in ways that are similar to ships floating on the water. The Quanser ALTAV [1, 3] provides a platform to demonstrate control reallocation methods for unmanned aerial vehicle. Fig. 4 shows the ALTAV UAV figures.
Figure 4. ALTAV UAV

ALTAV system used is a six degrees of freedom unmanned aerial vehicle. The variables describing the motion are $x$, $y$, $z$, $\theta$, $\gamma$ and $\phi$. These variables correspond to the translation in $x$, $y$ and $z$ directions and rotation about $z$, $y$ and $x$ axes (heading, pitch and roll), respectively. It should be noted that the system uses a ‘right-hand’ coordinate system with the positive $z$ direction as down. The behavior of the ALTAV system is governed by the following governing six equations (in the vehicle frame):

\[
\begin{align*}
M\ddot{x} &= F_i \sin(\gamma) - C_x \dot{x} \\
M\ddot{y} &= F_i \sin(\phi) - C_y \dot{y} \\
M\ddot{z} &= -F_i \cos(\gamma) \cos(\phi) - F_B + M_z - C_z \dot{z} \\
J_\phi \ddot{\phi} &= (F_i l - F_2 l + F_3 l + F_4 l) \sin(\rho) - C_\phi \dot{\phi} \\
J_\gamma \ddot{\gamma} &= (F_i l - F_3 l) - F_B L_B \sin(\gamma) - C_\gamma \dot{\gamma} \\
J_\phi \ddot{\phi} &= -(F_2 l - F_4 l) - F_B L_B \sin(\phi) - C_\phi \dot{\phi}
\end{align*}
\]

Symbols on behalf of, see nomenclature.

ALTAV simulation is designed to permit the examination and evaluation of candidate control algorithms for the operation of one or more such vehicles. The ALTAV simulation diagram can be divided into 4 distinct regions: the flight path input, the flight command controllers, the physical model of the system, and the “Real World” correction. Flight path block refers to as trajectory generators, generate a desired flight path for the vehicle. Currently the vehicle’s flight paths are specified by $(x, y)$ coordinates, elevation and vehicle heading.

In addition, each actuator has a rate limiter and a saturation cap. These values have been selected based on empirical tests performed on the system components such as maximum thrust and maximum vectoring rate.

Faults in control motors are implemented in the Simulink model.

V. Simulation Results Using WLS Method

In this section, partial loss and stuck at unknown position in the motor control surface and the reconfigurable control system is implemented in ALTAV non-linear model. Evaluation of the influence of Gaussian noise is given first, then trajectory selection, finally simulation results are given in partial loss and stuck failures.

A. Influence of Gaussian Noise

In order to evaluate the performance of the system in the real world, it is necessary to model such efforts as sensor noise and sensor delay. This block applies Gaussian noise to each of the system state variables with some predefined delay. The noise is assumed to have zero mean and bias with a variance determined either theoretically or through experimentation.
In practical systems, different kinds of disturbances always exist, and sometimes the influence of disturbance occupied an important role during the system operation. In this paper, Gaussian noise is added into simulation models although reconfigurable models with Gaussian noise take time to operate, simulation is slow.

B. Trajectory Selections

There are two different trajectories used as control input commands for the ALTA V platform. Flight path block refers to as trajectory generators which generate a desired flight path for the vehicle to follow. The flight path is a sequence of time-stamped position. Currently the vehicle’s flight paths are specified by (x, y) coordinates, elevation and vehicle heading. In this paper there are two different trajectory selection design, squares trajectory and circle trajectory as the desired flight paths, respectively. Square trajectory path is currently specified through the variables defined in an M file, circle trajectory path is designed through block diagram. Of course this information can be easily supplied from other sources, such as variables from the workspace or other generated flight paths. See Fig. 5.

Fig. 6 and Fig. 7 show the virtual trajectories with disturbance shake around the desired trajectories, its deflection error is bigger than the virtual trajectory without Gaussian noise, but its tendency still tracks the square or circle trajectory.

C. Partial Loss Simulation Results

The following simulation scenario has been carried out: motor 1 has 80% loss, square trajectory as commanded input. Simulation time is 80 second, the fault occurs at 50 second and the control reconfiguration is assumed to start at 50 second without time delay in FDI; the simulation result using WLS method is shown on the right.

The following Fig. 8 shows the UAV virtual tracking trajectory with/without re-allocation under motor1 control effectiveness factor $\gamma=0.20$. Comparisons were made under normal situation, without control re-allocation, and with control re-allocation method. In this scenario, it can be clearly seen, without control reallocation, UAV cannot track the desired trajectory, its magnitude error increases following the fault time. With control reallocation, UAV virtual trajectory can track desired trajectory with steady magnitude error. Different levels of partial faults occurred.
in motor 1 or other motors are also simulated and tested in ALATV non-linear model.

![Graph](image1)

**Figure 8. UAV virtual tracking trajectories with/without reallocation under partial loss**

D. Stuck Failure Simulation Results

Circle trajectory is used as commanded input, and motor 2 stuck at 0.8 is simulated in this scenario. Simulation time is 120 second, the fault is generated at 80 second and the control reconfiguration starts at 80 second.

The above Fig. 9, curves include normal situation, without control re-allocation, and with the control re-allocation method. To compare the performance of these three trajectories in this scenario, UAV virtual trajectory can track the desired trajectory well with control reallocation method whereas fails to follow the desired trajectory without control reallocation, its magnitude error increases after fault occurrence. Different levels of stuck failures occurred in motor 2 and other motors are also simulated and tested in ALATV non-linear model. Those results are not presented in this paper due to space limit.

![Graph](image2)

**Figure 9. UAV virtual tracking trajectories with/without reallocation under stuck failure**

VI. Conclusion and Future Work

The performance of UAV reconfigurable control system is investigated under the presence of partial and stuck faults. Detailed description about the control re-allocation technique has been demonstrated. Weighted Least Squares (WLS) method has been implemented and tested under the nonlinear ALTAV UAV benchmark model. Control reallocation techniques, carried out when faults occur, are analyzed and simulation results have demonstrated the effectiveness of the proposed techniques. WLS method shows an effective tracking of commanded input in the form of partial loss and stuck failure. As one of the future works, further improve the performance of WLS method. In addition, other new types control reallocation methods are also to be investigated and compared with the WLS method investigated in this paper. For instance, as proposed by D. Theilliol [28], Weighted Matrices $W_u$ and $W_v$, defined in Eq. (7), could be synthesized on-line based on the reliability of the components in order to guarantee the safety of the reconfigured system.

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