Hierarchical strategies of optimization for structural system identification based on the condensation method

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In the system identification using finite element method (FEM), system responses of overall degree of freedoms (DOFs) are necessary. Because of the limitation of sensor and other experiment equipment, the responses of unspecified DOFs have to be contained in the design variables. This increase of the design variable makes difficult to solve the inverse problem. It is one of the solutions about the problem of limited responses that the responses of unspecified DOFs are represented by the responses of specified DOFs, using the condensation method. In the previous study, we applied iterative inverse perturbation method (IIPM) to enhance the efficiency and the solution convergence of the structural system identification problems using the condensation method. So we efficiently identify structural system through solving the optimization problem with design variables which have the same number of elements. However, if the size of problem developed to analyze practical model is increased, the number of design variables which had to be considered in solving process is extremely increased. To identify large structural system, optimization strategy to efficiently change design variables during the iteration of optimization is required. In this study, we suggest two optimization strategies which are adaptive sub-domain method and genetic concept method. Numerical examples are presented to verify the efficiency of the proposed methods and to compare with those methods.

I. Introduction

RECENTLY the finite element method has been used in the various fields. However, even increasing capabilities of digital computers and developing methodology of FEM, the automated application of FEM to inverse problems has been limited by the numerous difficulties in mathematical formulation and computational resources. Especially, in the system identification using FEM, the system responses of overall DOFs are necessary. But, in general, overall the responses of real system cannot be measured by experiment due to the limit of sensor and other experiment equipment. The responses of unspecified DOFs have to be contained in the design variables. This increase of the design variable causes increasing the calculation time and make difficult to solve the inverse problem.

Using the condensation method, the responses of unspecified DOFs can be excluded in the design variables. The condensation method is classified into two-category, DOFs-based condensation and mode-based condensation. DOFs-based condensation method has an advantage in the system identification which can repeatedly use the position of primary DOFs. In the DOFs-based condensation method, mode shapes of secondary DOFs are represented by mode shape of primary DOFs using transformation matrix. Using this relation, the responses of unspecified DOFs can be represented by the responses of specified DOFs. This transformation can exclude unspecified DOFs in the design variables, but generates error. In the inverse problem, minor errors in transformation can impair the inverse iterations and decrease the convergence of solution. In this study, we reduce the error of transformation that is associated with system reduction, using the iterated improved reduced system (IIRS) 6 which is one of DOFs-based reduction method.

In the system identification using condensation method, the primary DOFs are directly related to the position of the sensor in the experiment of real system, and the solution accuracy and convergence of the reduced system depend upon the proper selection of the primary DOFs. The sequential elimination method (SEM) ⁹ is one of the most reliable selection methods. But the SEM requires considerable computational time in elimination procedure.

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To overcome disadvantage of SEM, the Two-Level Condensation Scheme (TLCS) was proposed by Cho and Kim\textsuperscript{10-12}. The TLCS does not require much computational time than SEM because of the estimation of the energy at the element-level\textsuperscript{11}. And, the TLCS was combined with a sub-structuring method\textsuperscript{14} for efficiency of solution.

In previous study, we perform system identification through solving the optimization problem with design variables that represent only change of element matrices, without the responses of unspecified DOFs, and error in the transformation matrix can be efficiently reduced, through implementation of the reduced system method using TLCS and IIRS. However, if problem size which developed to analyze practical model is increased, the number of design variables which had to be considered in solving process is extremely increased. To identify large structural system, optimization strategy to efficiently change design variables during the iteration of optimization is required. In this study, we suggest two optimization strategies which are adaptive sub-domain method and genetic concept method. In final draft, numerical examples are demonstrated to verify the efficiency of the proposed methods and to compare with those methods.

II. Inverse Perturbation based on the condensation method

In the general dynamic characteristics problem, the baseline system is presented as Eq. (1). In this paper, the baseline system means initial FE model.

\[ K\phi - \lambda M\phi = 0 \]  

Perturbed system is presented as Eq. (2). In this paper, the perturbed system means the updated FE model which has same response of real system.

\[ K'\phi' - \lambda' M'\phi' = 0 \]  

Eq. (2) can be expressed in the form of residual error.

\[ R = K'\phi' - \lambda' M'\phi' \]  

The structural changes between the FE model and the real model can be expressed perturbation, $\Delta K$ and $\Delta M$.

Using the perturbation, Eq. (3) is presented as Eq. (4). To calculate the structural changes, $\Delta K$ and $\Delta M$, is the object of structure system identification base on the FE method.

\[ R = ((K + \Delta K)\phi' - \lambda'(M + \Delta M))\phi' = (K - \lambda' M)\phi' + (\Delta K - \lambda'\Delta M)\phi' \]  

The structural changes, $\Delta K$ and $\Delta M$, are determined through the minimization of the residual which given in Eq. (4). In this state, $\phi'$ and $\lambda'$, are regarded as given values from the responses of the real system. In general overall responses of the real system $\phi'$ cannot be measured by experiment due to the limitation of sensor and other experiment equipment. The response of total DOFs is presented by the specified responses ($\phi_p'$), using transformation matrix in the condensation method which described in Eq. (5). But, the transformation matrix generates error. In the inverse problem, minor errors in transformation can impair the inverse iterations and decrease the convergence of solutions. In this study, we reduce the transformation error, using the IIRS.

\[ \phi' = \begin{bmatrix} \phi_p' \\ \varphi' \end{bmatrix} = \begin{bmatrix} I \\ \tau(\alpha) \end{bmatrix} \phi_p = T(\alpha)\phi_p' \]  

The residual vector (4) is formulated as a simple form in Eq. (6). The structural changes are exactly expressed as functions of structural parameters, $\alpha_c$. 

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The proper selection of the primary DOFs has a significant effect on the convergence of the inverse solution. The TLCS combined with sub-structuring scheme is used to prevent disadvantages of single-domain approach, like as that the PDOFs might be localized which results in excessive emphasis on lower modes or the loss of important modes.

III. Adaptive sub domain method

A. Adaptive sub-domain method

In adaptive sub-domain method, FE model is divided by several sub-domains. Each sub-domain has one design variable which represents structure change between FE model and real system. We operate the optimization for system identification until obtained just the pattern of each design value of sub-domain. Using this pattern, we divide or discard each sub-domain. Sub-domains are divided or discarding using relative ratio of maximum design variable in the design variables of sub-domains that consist of more than two elements. If maximum design variable was defined the maximum value in the design variables of total sub-domain, maximum design variable should be similar real structure change, regardless of iteration. So, the criterion is dependent on some elements which have large structure change. Though some elements which have large structure change, the sub-domains which have a relative small structural change could be excluded. So, maximum design variable is defined the maximum value in the design variable of sub-domains that consist of more than two elements.

As shown in figure 2, sub-domains that have value higher than lower limit of division level (Ldivide) are divided, and sub-domains that have value lower than upper limit of discard level (Udiscard) are discarded. Ldivide and Udiscard are percent parameters of adaptive sub-domain method. Though the repeated division and discard, we search the location of elements which have structure change between FE model and real system. Then the size of structure change is effectively obtained, using optimization with only the design variable of elements which are expected to have structure change.

B. Restoration of discard level

As mentioned previous section, the elements of sub-domains, which have value lower than Udiscard, are discarded from design variable in adaptive sub-domain method. When system has large structure change, this step has possibility of exception of some structure change. In this case, we cannot obtain the exact solution, because the design variable do not contains all structure change. So, if exact solution cannot be calculated in the final optimization, the sub-domains in the discard level are changed to a division level. From next step, the sub-domains which were in the discard level are divided, same as other division level, to find structure change. Even if some structure changes are discarded in the design variable, it is contained though restoration of discard level. Figure 2 is shown the schematic configuration of adaptive sub-domain method.
IV. Genetic concept method

A genetic algorithm (GA) is one of global search heuristics. GA is a search technique used in computing to find exact or approximate solutions to optimization and search problems. GA is implemented in a computer simulation in which a population of abstract representations of candidate solutions to an optimization problem evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s. But if we only use GA method, then we require to the big chromosome that causes consuming large computer resource. So, we suggest the genetic concept method that includes GA step between the iterations using gradient-based method. The genetic concept method searches the location of structure change using genetic algorithm and calculates the size of structure change using gradient-based method. As shown in figure 3, First we identify with design variable has same number of elements in first several iterations. Then population is generated using probability derived from structural change in the first several iterations. Each element has the probability of existence of structure change which described in Eq. 2. $\alpha_e$ is the value of design variables of element. $\alpha_{max}$ is the largest value of design variables in all elements. $P_{\text{mutation}}$ is a probability of mutation. It is 100 percent that probability of element which has largest value of design variables in all elements. Elements with zero value of design variable only have a probability of mutation. Population, in GA step, is generated using probability of existence of structure change, $P_e$.

$$P_e = \frac{\alpha_e}{\alpha_{\text{max}}} + P_{\text{mutation}}$$  \hspace{1cm} (7)

Next step is determent the fitness of each individual. The fitness is the difference of the residual error of present iteration and the residual error of next iteration when each chromosome is applied. And the best individual that has lowest residual error in the all chromosomes is selected. If finesses are same, chromosome which has a small number of design variables is selected. And the structural change is calculated with selected individual, using gradient method. Next generation is generated using probability derived from structural change in the previous step.

V. Numerical Example

In this section, we examine the efficiency of the adaptive sub-domain method and the genetic concept method, through numerical examples of damage detection. The example is that of a cantilever plate. As shown in figure 4, the total number of elements is 66, and the total number of nodes and DOFs are 84 and 504, respectively. The method for selecting PDOFs is TLCS. In figure 4, the nodes marked with blue circles represent the positions of PDOFs, while perturbations are applied for the elements marked with stars. We assume the damage distribution as shown in figure 5. Table 1 shows the results of eigenvalue analysis of the baseline system and the perturbed system though the damage distribution. The magnitude of change between the baseline system and the perturbed system is under 5%. From the modal response of the perturbed system, the inverse problem is solved.
Figure 6 shows progress step using adaptive sub-domain method. Upper figure means sub-domain distribution in the structural system. Lower figure shows the design variable in each element. Z-axis means damage percent, and color means sub-domain number. Right bar shows the number of sub-domain and each sub-domain color. Figure 7 shows progress step using genetic concept method. First, we obtain the design variable thought optimization with design variable has same number of elements in first several iterations as shown by figure 7(c). Then population is generated like figure 7(a). Figure 7(b) shows residual error of each chromosome, and figure 7(d) shows result after gradient method step with selected chromosome.

Table 1. Natural frequency comparison

<table>
<thead>
<tr>
<th>#. Mode</th>
<th>Baseline Sys.</th>
<th>Perturbed Sys.</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.179</td>
<td>0.178</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>3.188</td>
<td>3.160</td>
<td>0.89</td>
</tr>
<tr>
<td>3</td>
<td>7.006</td>
<td>6.983</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>34.02</td>
<td>33.69</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>55.54</td>
<td>54.93</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Figure 6. Progress step using adaptive sub-domain approach

Figure 7. Result of numerical example, using genetic concept method
Figure 8. Comparison of function call and time

Figure 9. Damage distribution of numerical examples

Figure 10. Result of numerical examples
Figure 8 shows the comparison of calculation time. In non-optimization strategy case, reduction method is only applied without any optimization strategy. As shown in figure 8, it is observed that adaptive sub-domain method and genetic concept method are more efficient than non-optimization strategy. It should be mentioned that two hierarchical strategies are applicable to the practical problems of damage detections or system identifications with efficiency.

To show the change of efficiency by damage distribution, we perform system identification with four the numerical examples which have different damage distribution as shown figure 8. Figure 9 shows the result and calculation time of numerical examples using proposed methods. In all damage cases, the calculation times of proposed methods are shorter than the calculation time of non-optimization strategy case, and calculation time of adaptive sub-domain method is changed by damage distribution. Because, when system has large damage, adaptive sub-domain method has possibility of exception of some damage. In this case, restoration of discard level is generated to contain the excluded damage. So, the sub-domains which were in the discard level are divided, same as other division level. This restoration of discard level spends the calculation time. Otherwise genetic concept method makes long time in the first full iteration, but has similar calculation times in the numerical examples.

VI. Conclusion

In this paper, we suggest two hierarchical strategies of optimization for structural system identification based on the condensation method. The first method is adaptive sub-domain method. The adaptive sub-domain method separates FE model by several sub-domain. Though the repeated division and discard process, we include only elements which are expected to have structure change in the design variables. Then, the size of structure change is effectively obtained, using optimization with only the design variable which have elements are expected to have structure change. Even if some structure changes are discarded in the design variable, they are contained through restoration of discard level. The second method is genetic concept method. The genetic concept method execute GA step between the iterations of gradient method, with chromosomes which represent the existence of structure change in all elements. Population is generated using probability derived from structural change of the gradient method step. The chromosome which has best fitness is selected and the structural change is calculated with the selected individual, using gradient method. We verify the efficiency of the proposed methods and compare these methods through numerical examples of damage detection. In all damage cases, the calculation times of proposed methods are shorter than the calculation time of non-optimization strategy case. It is observed that two proposed methods are more efficient than non-optimization strategy. The calculation time of adaptive sub-domain method is changed by damage distribution, more than calculation time of genetic concept method. Genetic concept method makes long time in the first full iteration, but has similar calculation times in various damage distribution cases. We think that these methods are helpful to identify the large structural system, and can be applied in the any other inverse problem also.

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References


