

## ANNOTATION

of the PhD Thesis on the Specialty 6D060400-Physics of  
Myrzakulov Nurgissa Ansatbayevich  
“Cosmological models of early and late Universe with bradyon and  
tachyon fields”

**Topicality of the research.** Currently, the major unsolved problems of modern physics can be attributed the problem of the early and late epoch of evolution of the Universe. Nowadays it is assumed that the Universe appeared 15-20 billion years after the Big Bang, when it started to expand with acceleration. This period of the evolution of the early Universe is known as the inflationary period of the expansion of the Universe, corresponding to the first moments of its birth and since then it has been continuously expanding and cooling.

The formulation of the Big Bang model was started in the 1940s with the work of G. Gamow and his colleagues R. Alpher and R. Herman. For the assumption that all particles of our Universe have a cosmological origin, they suggested that the early Universe, which was once a very hot and dense, expanded and cooled to its present state, the consequence of which had to be the presence of the relic background radiation with a temperature of the order of several Kelvin, which really was discovered after 16 years as the microwave background radiation. However, this did not explain neither the problem of large-scale homogeneity and isotropy of the Universe nor the problem of a flat Universe, when according to the observations, the average density of the Universe is close to the critical density.

To solve these problems in 1981 A. Guth proposed the inflation model, which was later developed by A. Starobinsky., A. Linde., B. Mukhanov and others. At present, this model is considered as the most likely candidate for the role of theory which explains the above mentioned problems of the Big Bang model. But there are problems with the discovery of the causes that trigger the inflation mechanism. In addition, as we know, after the period of inflation, the Universe will be empty and all matter in the Universe are born as a result of quantum processes. So, the Galaxies were formed from quantum fluctuations, produced in the late stages of the inflationary epoch. In the early Universe the wavelengths of the quantum fluctuations of the scalar field increased exponentially. When the wavelength of the fluctuation becomes greater than  $H^{-1}$ , it ceases to oscillate, and its amplitude slows down to a non-zero value  $\delta\phi$  due to the viscous term  $3H\dot{\phi}$  in the equation of the gravitational field. Subsequently, the amplitude of the fluctuation is almost unchanged, while its wavelength grows exponentially. In this context, it is expected that the quantum vacuum effects or gravitational quantum effects of gravity should play an important role in cosmological evolution of the early time evolution of the Universe. It has been shown that the quantum effects due to the conformally invariant fields (conformal or trace anomaly induced by an effective action), will be the most relevant in the early Universe, and can lead to the de Sitter solution. Recent cosmological data show that the trace anomaly, or closely related to it,  $R^2$  and  $T^2$  gravity can be one of the most realistic candidates

for the inflationary theory. Such modification attracts a number of researchers to study the various aspects of  $R^2$  and  $T^2$  inflation.

After the inflationary period, the epoch described by the Friedmann model, where the four-dimensional space-time is considered as a flat, homogeneous and isotropic, begins. Such homogeneous and isotropic space-time make it possible to extend the Copernican principle to the cosmological principle, stating that all spatial directions and some large enough areas in the Universe is essentially equivalent.

But at the end of 20th century the joint analysis of experimental data Wilkinson Microwave Anisotropy Probe (WMAP), and the results of observational data of Supernovae Type Ia strongly indicated that the Universe is currently expanding with acceleration. This effect is called the inflation of the late time Universe. This observing accelerated expansion states that currently the universe is dominated by the so-called dark energy, which can be understood as uniformly distributed substance with negative pressure. The issue of the dark energy, or why the current universe is expanding with acceleration, is considered as one of the most fundamental problems in modern physics.

One of the methods of invoking dark energy into the general theory of relativity in a cosmological context is the cosmological constant  $\Lambda$  introduced by Einstein, which is a constant physical quantity that characterizes the properties of the vacuum, where the field equations admit a spatially homogeneous statistical solution.

Among other methods of solving the problem of dark energy scalar (quintessence or phantom) model, dark fluid with a complex equation of state, more complex field theory with fermions, Abelian or non-Abelian vector field, string theory / M-theory, a space of higher dimension and etc. can be mentioned. Despite several attempts, there is still no any satisfying explanation of the origin of the dark energy. This is understandable, meaning that the current values of the cosmological parameters have not been determined with sufficient accuracy yet, and even less is known about their evolution.

To clarify the types of cosmic matter in most cases, the relationship between pressure and energy density, so-called the equation of state parameter written in the following form, is used

$$\omega = \frac{p}{\rho}$$

where  $\omega$  - the equation of state parameter. According to recent astrophysical observations, the equation of state parameter for dark energy approaches to  $\omega = -1$ , and probably belongs to the interval

$$\omega = -1.13_{-0.25}^{+0.23}$$

This interval equation of the state parameter emphasizes essentially three different variants of values  $\omega > -1$ ,  $\omega = -1$  and  $\omega < -1$ .

Modification of theory of gravity is an alternative of introducing various fields in the action, describing the evolution of our Universe. One of the most successful examples of modified theory of gravity, for example, is considered to be well explaining the early time of inflation, Starobinsky model or  $R^2$  - gravity.

Teleparallel gravity corresponds to the gauge theory for the translation group (parallel translation). For the first time the theory of teleparallel gravity was proposed by Einstein himself. Due to the specific nature of translations, any gauge theory, including there transformations, will be different in many respects from ordinary internal gauge models, and the most significant is the presence of tetradof field. On the other hand, the tetrads of field, naturally, can be used to determine the linear Weitzenbock connection, which is the connection determined by the torsion not by curvature of space. The tetrad field can also be naturally used to determine the Riemannian metric in context of which Levi-Civita connection can be built. It is important to note that the curvature and the torsion are connection property, and different connections can be defined in the same space. Thus it can be said that the presence of a non-trivial tetrad field in the gauge theory induces both teleparallel and Riemann structure in the space-time. The first is connected to the Weitzenbock connection, and the second to the Levi-Civita connection. Due to the universality of gravitational interaction, you can link these geometric structures in the gravity.

In the context of teleparallel gravity curvature and torsion are able to provide equivalent descriptions of the gravitational interaction. But there are conceptual differences between them. According to general relativity, the curvature is used for geometrization of the space-time, and thus successfully describes the gravitational interaction. On the other hand, in the teleparallel gravity, torsion is an attribute of the gravity, in this case the curvature takes into account the gravity, not geometrizing the interaction, but acts as a gravitational forces.

In 1970 H.A. Buchdahl proposed to modify the Einstein-Hilbert action by replacing the lagrangian through the function of the scalar curvature in the form  $F(R)$ . This modified theory of gravity was used to describe inflation, dark energy, cosmological perturbations, spherically symmetric solutions. To derive the equations of motion in  $F(R)$  gravity, there are two gravity formalisms. The first is the standard metric formalism in which the field equations can be determined by variation of action with respect to metric tensor  $g_{ik}$ . In this formalism, the affine connection  $\Gamma_{jk}^i$  depends on the metric tensor  $g_{ik}$ . In Palatini formalism, in which  $g_{ik}$  both  $\Gamma_{jk}^i$  are considered as independent variables in variation of the action, these two approaches lead to different gravitational field equations for nonlinear Lagrangian in terms  $R$ , while for the action of general relativity, they are identical to each other.

On the other hand, over the past two decades there have been many attempts to modify the theory of gravity in order to be able to describe the evolution of the observable Universe, as well as alleviate the problems of renormalization of the theory of gravity. Most studies in the literature are formulated based on the curvature of space, as well as on modification of the Einstein-Hilbert action. Nevertheless, referring to this discussion, it is wise to start from teleparallel

equivalent of the general relativity, and use it as a basis for building a gravitational modification. The simplest of such modifications is  $F(T)$  gravity.

The key issue is that the teleparallel equivalent of the general relativity coincides with the general relativity at the equations level, but  $F(T)$  gravity is different from  $F(R)$  gravity as the gravitational field equations of  $F(T)$  gravity are the second order differential equations, while the field equations of  $F(R)$  are of the fourth order. The modified gravity has interesting cosmological solutions that provide an alternative interpretation of the accelerated expansion of the Universe.

One of the most interesting and promising versions of the modified theory of gravity is  $F(R,T)$  gravity, where  $R$  - a scalar curvature and  $T$  - scalar torsion. Thus  $F(R,T)$  theory of gravity is the union of  $F(R)$  and  $F(T)$  theories of gravity. In this case, we can take into account the contribution of each of the proposed models to gravity, which will give an opportunity to assess the degree of influence of the each.

In particle physics, all elementary particles are classified according to the velocity of their movement in the following way:

- bradions - particles that move slower than the speed of light in a vacuum and whose rest mass is nonzero. They consist of all known particles, except massless particles;

- luxons - particles that travel at the speed of light in vacuum and have no rest mass. These include photons, gluons and gravitons;

- tachyons - hypothetical particles moving faster than the speed of light in vacuum and having imaginary mass.

As bradion fields, in particular, consider ordinary scalar and fermion fields as containing matter. Recently, the scalar and fermion fields are actively investigated in cosmological applications. At the moment, the search of models that satisfactorily explain the past and present time of acceleration, is the subject of intense research. The most popular models are the scalar fields that are minimally or non-minimally coupled to the gravitational field. They play the role of the inflaton field or dark energy. Also fermion fields may be responsible for the accelerated periods with various modes. In some of these models fermion field plays a role of inflaton in the early period of expansion of the Universe and the dark energy for the recent scenario of expansion of the Universe without the need of a cosmological constant or a scalar field.

**The aim of the research and scientific outcomes.** To build and explore theories of new cosmological models of the early and current Universe in the framework of the modified gravity interacting with different types of matter. The results obtained in the thesis

- The role of the conformal anomaly in the induction of instability, which leads to inflation in  $F(T)$  modified model of gravity, is studied. It is shown that de

Sitter inflation can be implemented in the models  $F(T)=T^2 + AT + Be^{\frac{T}{T_0}}$ ,  $F(T)=T + T^2$  and  $F(T) = \beta T^n$  with cosmological constant. It has been shown that the inflation of the model is unstable due to the conformal anomaly. In the case

$F(T) = T + T^2$  to induce instability of the conformal anomaly, it is necessary that the cosmological constant is small.

– A model of  $F(T)$  gravity which is non-minimally coupled to the fermion fields for a flat, homogeneous and isotropic Friedmann - Robertson - Walker metric is investigated. Also  $F(T)$  model of gravity with scalar and fermion fields for the (2+1) dimensions was considered. By using the Noether symmetry approach exact cosmological solutions for this model are found.

– Modified mimetic  $F(R)$  gravity was considered, where the scalar curvature  $R$  depends on the metric tensor and scalar field. The equation of motion for scale factor and scalar field are derived. It is shown that the resulting de Sitter solutions are stable.

–  $F(R)$  gravity with  $k$  essence is studied. Non-minimal coupling between the gravity and field of matter was investigated. Einstein's equations for this theory are obtained. Particular cases for symmetry generators was considered and cosmological solutions are obtained.

– By using Noether symmetry approach the Lagrangian for  $F(R, T, X, \phi)$  models of gravity is reconstructed.

– The system of tachyon field non-minimally coupled with massive neutrino matter is examined. For this system, we examined the potential of the form  $V = \phi^{-2}$ , so that the tachyon field demonstrates scaling behavior. Also the case of the exponential potential inspired by string theory is also considered, which does not lead to a scaling solution for this model. In this case, a solution similar to models of dark matter, is an attractor of late time system.

**The object of the research.** The evolution of the Universe in different cosmological models.

**The subject of the research.** Cosmological solutions of gravitational equations with matter, describing the era of dominance of inflation and dark energy of the Universe.

**The scientific novelty.** The scientific novelty of the research is as follows:

- Model of the Universe in  $F(T)$  theory of gravity with conformal anomaly;
- Model of the Universe in  $F(T)$ ,  $F(R)$  and  $F(R, T)$  theories of gravity with scalar and fermion fields;
- Model of the Universe with tachyon fields non-minimally coupled with the massive neutrino matter.

**Research objectives.** The main objectives of the thesis are as follows:

- In framework of modified theory of gravity to obtain the cosmological solutions describing the early and late accelerated expansion of the Universe and investigate the effect of the conformal anomaly;
- In frame of modified  $F(R)$ ,  $F(T)$  and  $F(R, T)$  theory of gravity with scalar and fermion fields and using Noether symmetry method to show how the obtained cosmological solutions describe dark energy;
- In framework of mimetic  $F(R)$  theory of gravity to make sure that the de Sitter solution is stable;

- In Friedmann-Robertson-Walker space-time to consider tachyon field that is nonminimally coupled with massive neutrino matter. To compare these solutions with observational data.

#### **Statements for the defense.**

- In the framework of  $F(T)$  model of gravity the influence of the conformal anomaly on the stability of the de-Sitter solution for flat, homogeneous and isotropic Friedmann-Robertson-Walker space-time was shown. The instability of the solution, leading to the end of the inflationary period of the Universe, was revealed;
- The cosmological model of the late Universe in the framework of  $F(T)$  and  $F(R)$  modified theories of gravity with non-minimally coupled scalar and fermion fields. It was shown that such models describe the accelerated expansion of the universe, leading to a Big Rip;
- The model of tachyon field, non-minimally coupled with the massive neutrino matter. The solutions for the inverse square potential and exponential potential inspired by string theory. Normalized Hubble parameter was constructed and a comparison of the obtained results with observational data was made.

#### **Practical significance of the research outcomes.**

The results obtained in the thesis have theoretical character and can be used for constructing new cosmological models of the early and late Universe in framework of modified theory of gravity with a scalar - fermion fields and tachyon fields.

#### **Personal contribution of the PhD student.**

In process of implementation of the research under the supervision of the scientific advisors, author was directly involved in all stages of work: did all calculations, built the graphics of found solutions, personally prepared publications.

#### **Approbation of the thesis outcomes.**

The results obtained in the thesis have been discussed and presented at:

- 4th International Conference on Mathematical Modeling in Physical Sciences (IC-MSquare2015). Mykonos, Greece, 5–8 June 2015;
- International Conference, in honor of the 70-th anniversary of academician of NAS RK Takibaev Nurgali Zhabagaevich, February 21-22, 2014;
- IX international scientific conference of students and young scientists "Science and Education - 2014". April 11, 2014;
- XI international scientific conference of students and young scientists "Science and Education - 2016". April 14, 2016.

In addition, the results were presented and discussed at scientific seminars of the Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, on seminars of Eurasian International Centre for Theoretical Physics, seminars of Institute of Space Sciences (Spain) and Department of Physics, California State University (USA).

**Publications.**

According to the results of the thesis was published 10 publications, including 2 articles in international journals with high impact factor; 1 article was presented at 4th International Conference on Mathematical Modeling in Physical Sciences and was published in Journal of Physics: Conference Series; 3 articles in periodical editions recommended by the Committee for Control of Education and Science of the Ministry of Education and Science, Republic of Kazakhstan; 4 articles in the materials of international conferences in Kazakhstan.

**Volume and structure of the thesis.**

The thesis consists of an introduction, three chapters, conclusion and list of references with 290 titles, includes 102 pages of main computer text, including 6 figures and 2 tables.